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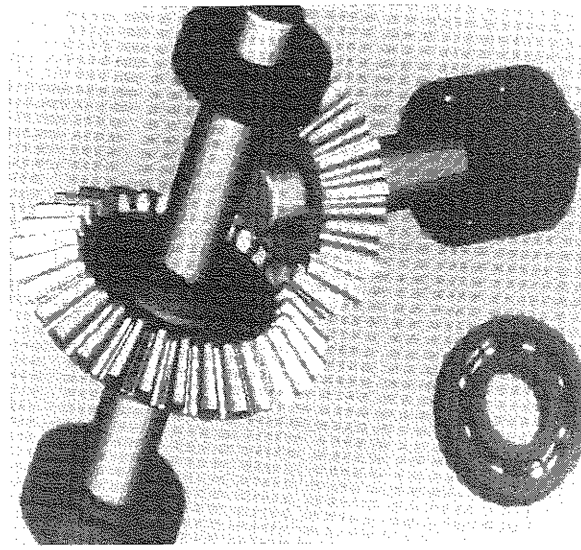

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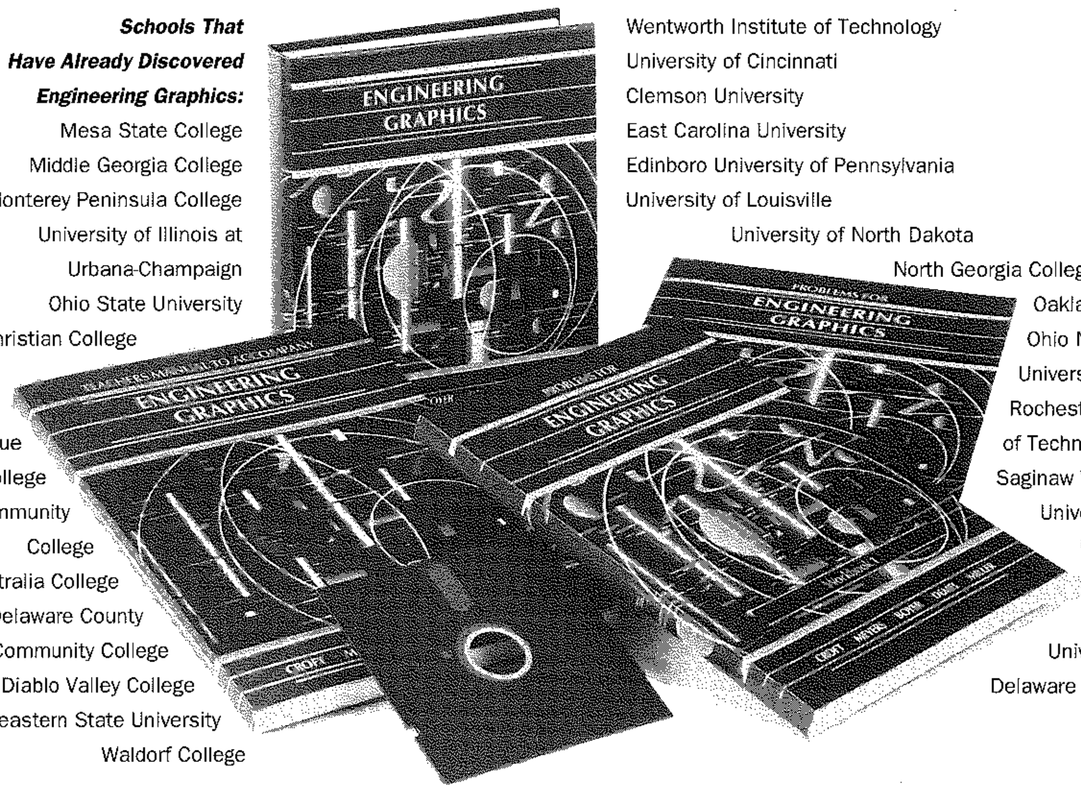

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Multiple Cubic Bezier Curves

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An efficient algorithm is described for generating smooth curves of first-order continuity. The algorithm, referred to as the multiple cubic Bezier curve, is composed of several cubic Bezier curves joined together at the user defined control points. The multiple cubic Bezier curve satisfies all of the user-defined control points except the first and last points which, in the method presented, are used to define the direction of the curve at the end points.

Introduction

As today's engineers are continually faced with the design and analysis of increasingly complex computer models, the need for advanced interactive computer graphics is becoming exceedingly more crucial. The resulting need for the visualization of a conceptual design has brought to the market a number of computer graphics software packages with various degrees of sophistication. These packages have greatly enhanced the productivity of CAD workstations enabling the engineer to design, simulate, and further refine the shape of the application model until an optimum design is obtained prior to building an actual prototype.

Computer representation of shapes requires a mathematical representation of a model that does not exist as a best fit, per se; the designer estimates the shape and refines it until a satisfactory shape is obtained. In this sense the designer performs the role of a stylist by generating and regenerating images of an object model. Besides the stylist's feel for the shape of the curve, the functional form needed for the design of an arbitrary curve is not a priori known. Thus, what designers need is a collection of methods that could enable them to construct a precise mathematical description of the object which can lead to a rapid construction of the model via a communication link to a numerically controlled machine. In fact, the ability to create a well-defined model which can be manufactured is the goal of a solid modeling system. However, most real objects are composed of free-form shapes, not just a combination of blocks, cylinders, and spheres. Hence, to render a model of complicated shapes, one needs to be able to construct free-form curves and surfaces.

An efficient algorithm is described for generating smooth curves of first-order continuity. The algorithm will be referred to as the multiple cubic Bezier curve which is composed of several cubic Bezier curves joined together at the user defined control points. The multiple cubic Bezier curve satisfies all of the user-defined control points except the first and last points which, in the method presented, are used to define the direction of the curve at the end points.

The multiple cubic Bezier curve exhibits some local controllability, i.e., if the position of a control point is modified, only the shape of the curve in the vicinity of that point may be affected without significantly altering the shape of the curve throughout.

Multiple Cubic Bezier Curve

Since the derivation of the algorithm for the multiple cubic Bezier curve is based on the general Bezier curve, an introduction to the Bezier curve must first be presented. The Bezier method is named after the French engineer, Pierre Bezier, who developed a method for modeling the outer shapes of Renault automobiles. The Bezier curve produces a smooth curve based on a set of parametric equations. A fundamental property of parametric curves is that their shape is a function of the relative position of their vectors and not the total set of points with respect to the coordinate system used. The shape of a Bezier curve is defined by the coordinates of a series of points known as the control points that define the so-called characteristic polygon to which the Bezier curve will be confined. In other words, the Bezier curve lies entirely within the convex hull of the characteristic polygon. The end points of the Bezier curves are in common with the vertices of the polygon so that the curve always passes through the first and the last control points. The intermediate control points influence the Bezier curve by pulling the curve toward themselves (Fig. 1). This pulling effect depends on the position of the points used in the controlling parametric equations. The controlling parametric form for the Bezier curve is given as:

$$P(t) = \sum_{i=0}^{n} P_i B_{i,n}(t) \qquad (1)$$

This equation defines a smooth curve for the n + 1 arbitrary control points specified. The bending function $B_{i,n}(t)$ is based on the Bernstein polynomials and is defined as:

$$B_{i,n}(t) = n!/[i!(n-i)!] \cdot t^i (1-t)^{n-i} \qquad t \epsilon [0,1] \qquad (2)$$

The parameter P in Eq. (1) is a vector-valued function and can be written in terms of its x-y components:

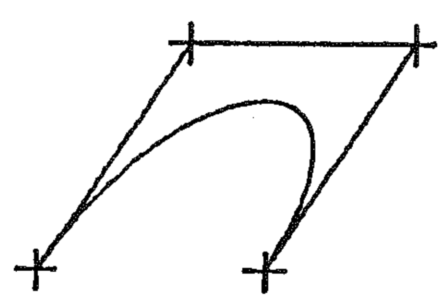


Fig. 1 Bezier curve and the characteristic polygon

$$x(t) = \sum_{i=0}^{n} x_i B_{i,n}(t) \qquad (3)$$

$$y(t) = \sum_{i=0}^{n} y_i B_{i,n}(t) \qquad (4)$$

The initial procedure for computing the coordinates of a Bezier curve requires calculating the blending function $B_{i,n}(t)$ for a given set of control points using Eq. (2). Next, computing x(t) and y(t) obtained from Eqs. (3) and (4) is required. A complex design can be achieved using this procedure by breaking the object into several parts, each of which is defined by a set of individual control points. The individual parts are then joined together at the end points. Continuity at the joint can be of order zero, one, or higher. Although a zero-order continuity at a joint can be easily implemented by making the end point of one curve coincide with the end point of another curve (Fig. 2a), it is of limited value. A more desirable continuity is that of first order which can be satisfied by requiring that the tangent vectors (slopes) match at the joint (Fig. 2b). A first-order continuity is used in further descriptions.

The multiple cubic Bezier curve consists of several cubic Bezier curve segments joined together end-to-end with a first-order continuity between the segments. To implement a first-order continuity, two additional control points are required. For example, for the three curve segments shown in Fig. 3, a total of six new control points, in addition to the six entered by the user (designated with a +) are necessary. In Fig. 3 these points are marked with a *.

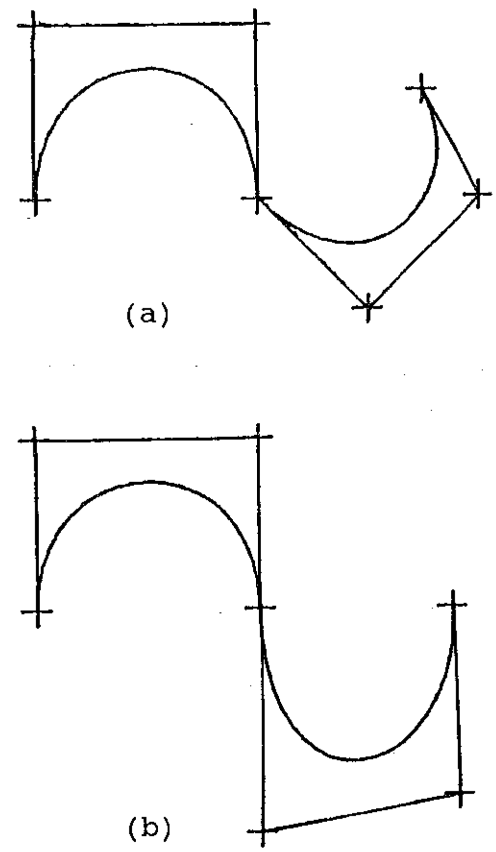


Fig. 2 Continuity of two classical Bezier curves
(a) zeroth order continuity
(b) first order continuity

To illustrate how the these new control points are generated, consider the three curve segments shown in Fig. 4. For curve II, the two additional control points R and S are placed along vectors B and C, respectively. Using points Q, R, S, and T, a cubic Bezier curve is drawn producing curve II.

Equations for computing the additional control points will be given with reference to Fig. 5, which is a magnified portion of Fig. 4. The following vectors are initially computed:

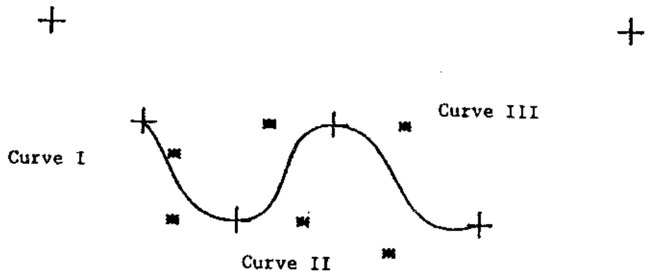


Fig. 3 Additional control points for the locally controlled Bezier curve

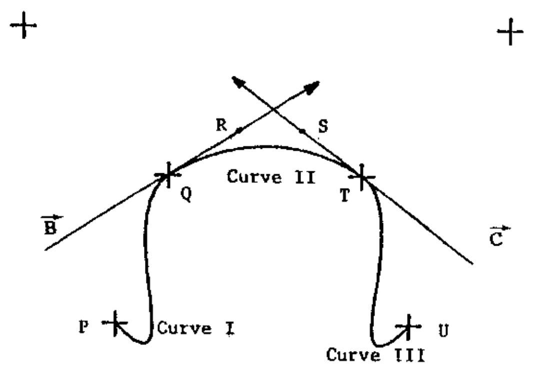


Fig. 4 Additional control points for curve II

$$\mathbf{A} = \mathbf{P}_2$$

$$\mathbf{B} = \mathbf{P}_1/|\mathbf{P}_1| + \mathbf{P}_2/|\mathbf{P}_2| \qquad (5)$$

$$\mathbf{C} = \mathbf{P}_2/|\mathbf{P}_2| + \mathbf{P}_3/|\mathbf{P}_3|$$

Next, the angle Θ is computed using the dot product relation of vectors **B** and **A**:

$$\text{Cos } \Theta = \mathbf{A}\cdot\mathbf{B}/(|\mathbf{A}||\mathbf{B}|) \qquad (6)$$

Then, the coordinates of R are computed from triangle QVR where point V is one-third the length of P_2. This one-third factor is chosen to evenly space the additional control points along the line P_2; however, it is worthwhile to note that this value essentially plays the role of a tension-control parameter. If this parameter is varied, different results may be produced. As illustrated in Fig. 6, if the tension control parameter is re-

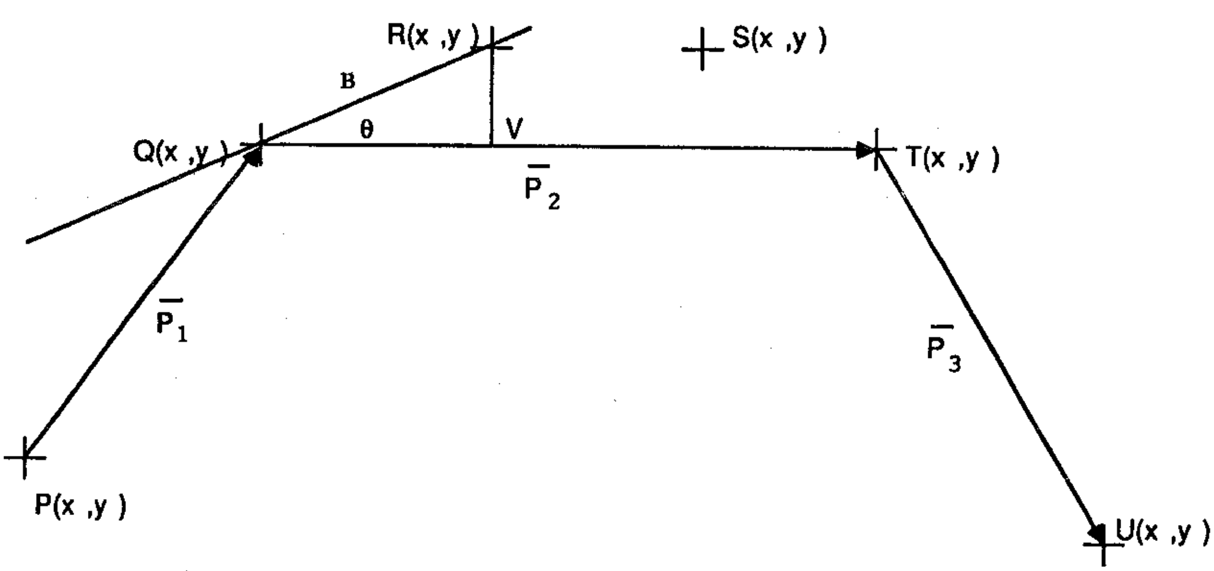
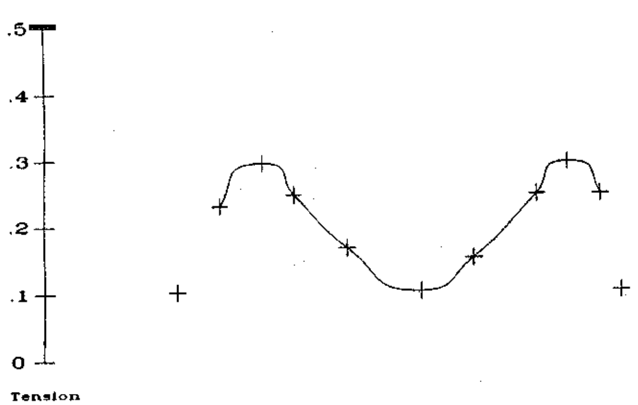
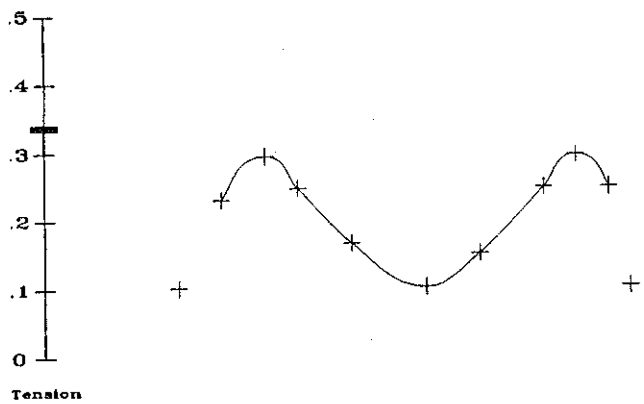


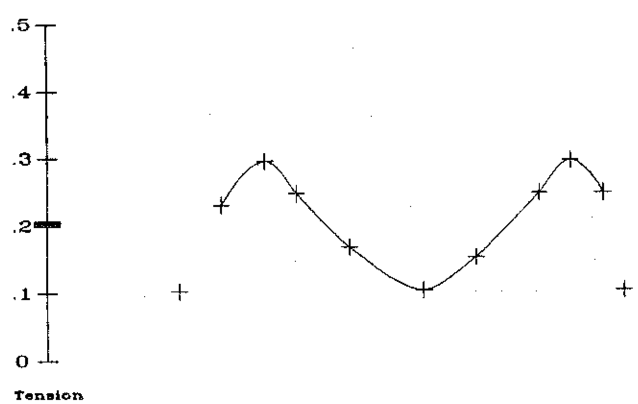
Fig. 5 A magnified portion of Fig. 4



(a) tension parameter = 0.5

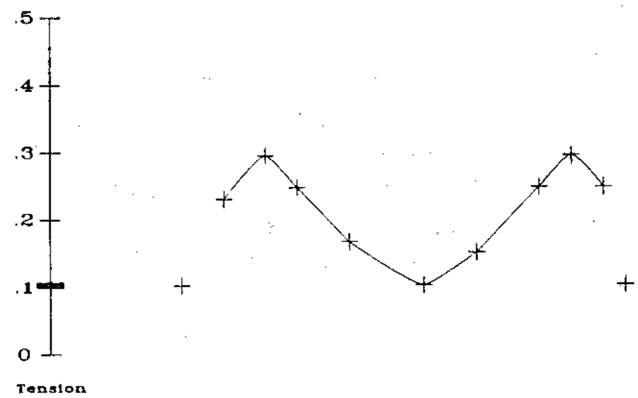


(b) tension parameter = 0.33

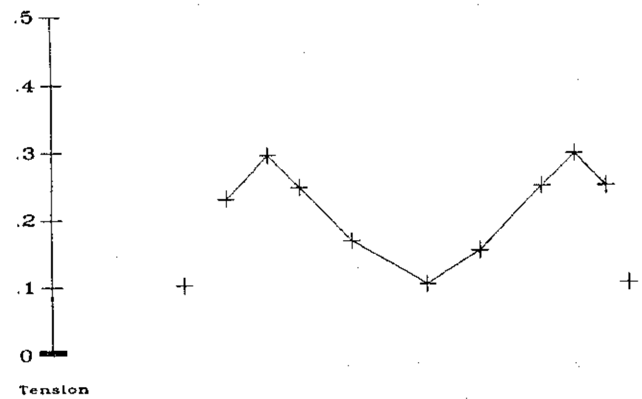


(c) tension parameter = 0.2

Fig. 6 Effects of tension-control parameter



(d) tension parameter = 0.1



(e) tension parameter = 0.0

Fig. 6 (continued)

duced, the curve produced essentially 'pulls' tighter toward the user-defined control points.

The length of the hypotenuse of triangle QVR may be determined using

$$L = 1/3|\mathbf{A}|/\cos\Theta \qquad (7)$$

Thus, the coordinates of the point $R(x'_i, y'_i)$ are:

$$x'_1 = B_x L + x_1 \qquad (8)$$

$$y'_1 = B_y L + y_1 \qquad (9)$$

where B_x and B_y are the x and y components of vector B. The ' is used to denote the coordinates of the new control point. Finally, substitution of Eqs. (6) and (7) into (8) and (9) yields the coordinates of the point R as given by Eqs. (10) and (11).

$$x'_1 = x_1 + B_x|\mathbf{A}|^2|\mathbf{B}|/3(\mathbf{A}\cdot\mathbf{B}) \quad (10)$$

$$y'_1 = y_1 + B_y|\mathbf{A}|^2|\mathbf{B}|/3(\mathbf{A}\cdot\mathbf{B}) \quad (11)$$

The coordinates of the control point $S(x'_2,y'_2)$ can be computed in a similar fashion. The results are:

$$x'_2 = x_2 + C_x|\mathbf{A}|^2|\mathbf{B}|/3(-\mathbf{A}\cdot\mathbf{C}) \quad (12)$$

$$y'_2 = y_2 + C_y|\mathbf{A}|^2|\mathbf{B}|/3(-\mathbf{A}\cdot\mathbf{C}) \quad (13)$$

Having computed the coordinates of the new control points R and S, the algorithm generates a smooth curve using the user defined control points, Q and T, and points R and S.

This procedure is combined with the next set of the user-defined control points until the curve is completed. Two sample outputs of the multiple cubic Bezier curve are presented in Fig. 7.

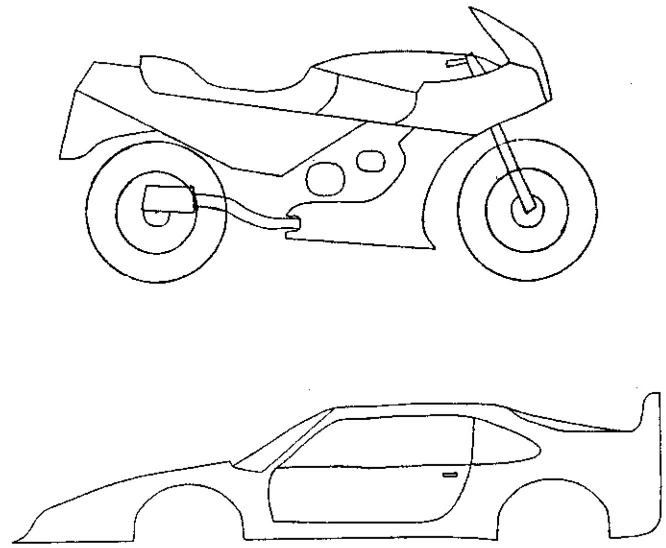


Fig. 7 Sample designs using multiple cubic Bezier curves

Concluding Remarks

A method for rendering smooth curves is presented which offers some improvements in controllability over the general Bezier curves in that the curve satisfies all of the user-defined control points except the first and last points. Furthermore, a tension control parameter is introduced which can be set by the user to control the tension of the curves, thus providing additional flexibility in the design of free-form curves. This method may be an alternative CAD tool since the shape of a portion of the curve can be easily adjusted without significantly affecting the entire shape of the design.

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A Solution to Linear Programming Problems by the N-dimensional Geometric Model

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Linear programming problems of n variables are described by the theory of n-dimensional descriptive geometry. A solution to linear programming problems by the n-dimensional geometric model is given. Several examples implemented on the computer are cited.

1. The N-dimensional Geometric Model of Linear Programming Problems

1.1 N-dimensional geometric interpretation of the mathematical model

In general, the mathematical model of a linear programming problem with n variables and m structural constraints can be expressed as follows:

$$\text{minimize } z = c_1x_1 + c_2x_2 + \dots + c_nx_n \tag{1}$$

subject to

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &\le b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &\le b_2 \\ &\dots \\ a_{m1}x_m + a_{m2}x_2 + \dots + a_{mn}x_n &\le b_m \end{aligned} \tag{2}$$

$$\text{where } x_1, x_2, \dots, x_n \ge 0 \tag{3}$$

From the theory of linear programming, the numbers x_1, x_2, ..., x_n that satisfy all the constraints of Eqs. (2) and (3) are said to be a feasible solution. The set of all feasible solutions is the region of feasibility and is a convex set; the feasible solution that minimizes the objective function (Eq. 1) is the optimal solution.

In n-dimensional descriptive geometry, each of the m equations (Eqs. 2) of structural constraint (taken with the sign of equality) represents a hyperplane E^{n-1} in E^n and non-negative constraints (Eqs. 3) represent n coordinate hyperplanes in the E^n orthogonal coordinate system[1]. These m + n hyperplanes constitute a convex hyperpolyhedron, denoted by Φ, which is the first angle of the E^n orthogonal coordinate system. The coordinate values in the first angle are positive[2]. The vertices of the convex hyper-polyhedron are the set of basic feasible solutions which correspond to the vertices and vice versa. The optimal solution lies in the basic feasible solution which is to be determined.

1.2 Graphic solutions to the n-dimensional geometric model

As an example, let n = 4 and m = 4 to illustrate the region of feasibility of a linear programming problem (Fig. 1) and to analyze the construction of the vertices of Φ.

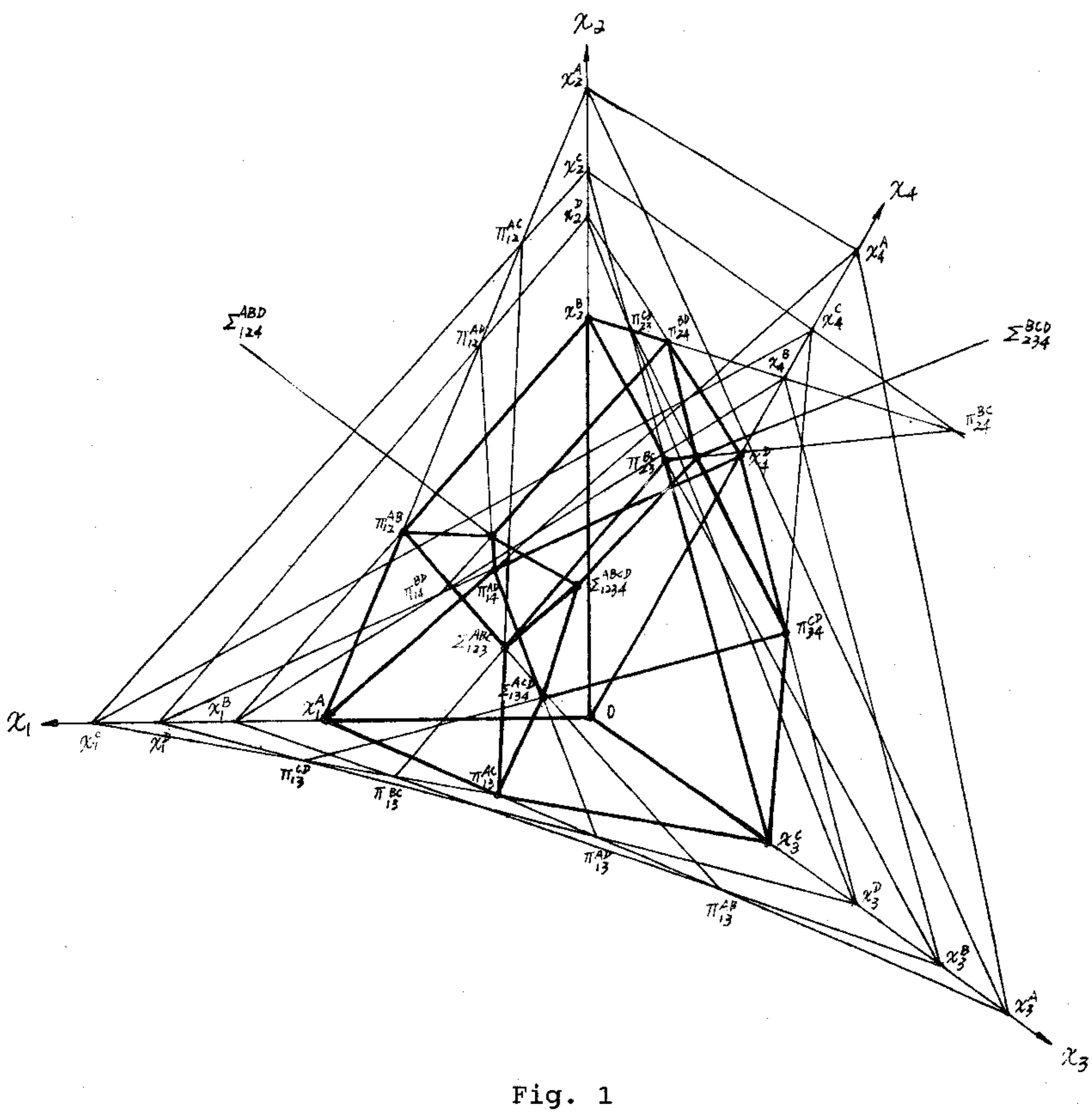

Fig. 1

Example 1

Required: Minimize $z = c_1x_1 + c_2x_2 + c_3x_3 + c_4x_4$

subject to

$$x_1/x^A_1 + x_2/x^A_2 + x_3/x^A_3 + x_4/x^A_4 \leq 1 \quad A$$
$$x_1/x^B_1 + x_2/x^B_2 + x_3/x^B_3 + x_4/x^B_4 \leq 1 \quad B$$
$$x_1/x^C_1 + x_2/x^C_2 + x_3/x^C_3 + x_4/x^C_4 \leq 1 \quad C$$
$$x_1/x^D_1 + x_2/x^D_2 + x_3/x^D_3 + x_4/x^D_4 \leq 1 \quad D$$

where $x_1, x_2, x_3, x_4 \geq 0$

Solution: Figure 1 shows the four-dimensional axonometric pictorial of the region of feasibility. Φ has sixteen vertices that are in distinct subspaces.

The notations of the vertices of Φ are as follows:

x^A_1 - the point that is generated on the axis x_1 by the hyperplane A.

π^{AB}_{12} - the point that is generated on the coordinate plane π_{12} by the hyperplanes A and B.

Σ^{ABC}_{123} - the point that is generated on the subspace Σ_{123} by the hyperplanes A, B, and C.

Σ^{ABCD}_{1234} - the intersection point of hyperplanes A, B, C, and D.

The notations of the remainder of the vertices are analogous.

The sixteen vertices of Φ in Fig. 1 are:

1) the origin (zero dimensional subspace), O,
2) the points on the coordinate axes (one-dimensional subspaces), x^A_1, x^B_2, x^C_3, and x^D_4,
3) the points on the coordinate planes (two-dimensional subspaces), π^{AB}_{12}, π^{AC}_{13}, π^{AD}_{14}, π^{BC}_{23}, π^{BD}_{24}, π^{CD}_{34},
4) the points on the three-dimensional subspaces, Σ^{ABC}_{123}, Σ^{ABD}_{124}, Σ^{ACD}_{134}, and Σ^{BCD}_{234},
5) the point on the four-dimensional subspace, Σ^{ABCD}_{1234}.

The vertices are constructed from the following operations:

$$0 = \Sigma_{123} \cap \Sigma_{124} \cap \Sigma_{134} \cap \Sigma_{234}$$
$$x^A_1 = A \cap \Sigma_{123} \cap \Sigma_{124} \cap \Sigma_{134}$$
$$\pi^{AB}_{12} = A \cap B \cap \Sigma_{123} \cap \Sigma_{124}$$
$$\Sigma^{ABC}_{123} = A \cap B \cap C \cap \Sigma_{123}$$
$$\Sigma^{ABCD}_{1234} = A \cap B \cap C \cap D$$

The remainder of the vertices are constructed in an analogous manner.

In Fig. 1, x^A_1 is the intercept point of the hyperplane A on the axis x_1; π^{AB}_{12} is the trace point of the intersection plane between hyperplanes A and B on the coordinate plane π_{12}, i.e., the intersection point of two trace lines of hyperplanes A and B on π_{12}; Σ^{ABC}_{123} is the trace point of the intersection line between hyperplanes A, B, and C on the subspace Σ_{123}. As shown in Fig. 1, the trace plane of the hyperplane A on the Σ_{123} is the quadripoint form $x^A_1\pi^{AC}_{13}\Sigma^{ABC}_{123}\pi^{AB}_{12}$; the trace plane of the hyperplane B is the quadripoint form $x^B_2\pi^{AB}_{12}\Sigma^{ABC}_{123}\pi^{BC}_{23}$; the trace plane of the hyperplane C is the quadripoint form $x^{C}{}_{3}\pi^{AC}{}_{13}$ $\Sigma^{ABC}{}_{123}\pi^{BC}{}_{23}$; $\Sigma^{ABC}{}_{123}$ is the common point of the above three quadripoint forms. In Σ_{123}, three intersection lines between hyperplanes A, B, and C are:

$$\pi^{AC}{}_{13}\pi^{AC}{}_{12} = A \cap C \cap \Sigma_{123}$$
$$\pi^{AB}{}_{12}\pi^{AB}{}_{13} = A \cap B \cap \Sigma_{123}$$
$$\pi^{BC}{}_{23}\pi^{BC}{}_{13} = B \cap C \cap \Sigma_{123}$$

In Fig. 1 because $\pi^{AC}{}_{12}$, $\pi^{AB}{}_{13}$, and $\pi^{BC}{}_{13}$ are not vertices of Φ, $\Sigma^{ABC}{}_{123}$ connects only with the three vertices $\pi^{AB}{}_{12}$, $\pi^{AC}{}_{13}$, and $\pi^{BC}{}_{23}$. Thus, if each two of three hyperplanes A, B, and C and Σ_{123} construct three vertices of Φ on the coordinate planes of Σ_{123}, $\Sigma^{ABC}{}_{123}$ is the vertex of Φ and one (only one) of two superscripts between any two of vertices $\pi^{AB}{}_{12}$, $\pi^{AC}{}_{13}$, and $\pi^{BC}{}_{23}$ is the same and the subscripts of three vertices contain three distinct coordinate axes.

The vertices of Φ (Fig. 1) contain three four-dimensional subspace points $\Sigma^{ABC}{}_{123}$, $\Sigma^{ABD}{}_{124}$, $\Sigma^{ACD}{}_{134}$, and $\Sigma^{BCD}{}_{234}$ and the four-dimensional space point $\Sigma^{ABCD}{}_{1234}$ is the intersection point of structural constraint hyperplanes A, B, C, and D. Hence, it connects only to those four three-dimensional subspace points.

Sixteen vertices construct the four-dimensional geometric model of the region of feasibility in Example 1.

2. Criteria for Vertices of Φ

2.1 $C^{n}{}_{m+n}$ intersection points distribute in E^n

In the n-dimensional geometric model of the linear programming problem above, constraint conditions contain m structural constraint hyperplanes and n coordinate hyperplanes (both taken with the sign of equality). The total is m + n hyperplanes with $C^{n}{}_{m+n}$ intersection points which are distributed as shown in Table 1.

In Table 1, the number of intersection points increases with increasing m and n. But fortunately, the number of vertices of Φ is much less than $C^{n}{}_{m+n}$. For example, while n = 4 and m = 4, the total of intersection points is 70 ($C^{4}{}_{4+4}$), but as shown in Example 1, the number of vertices is at most sixteen. Furthermore, after translating structural constraint hyperplanes to intercept form, the right-hand side may be of four forms: ≤ 1, ≥ 1, $= 1$, $= 0$; and the intercepts may be positive or negative. The chance that there is a vertex $\Sigma^{ABCD}{}_{1234}$ in Fig. 1 is rare.

2.2 Examples of determining vertices of Φ

Graphic solutions to two-dimensional linear programming problems have been described in numerous sources[3,4]. For simplicity, take n=3 and m=3 as examples.

Example 2

Required: For the general geometric model, minimize z = ...

subject to

Dimension $n=$		2	3	3	4	4	5	10	...
No. of structural constraints $m=$		3	3	4	4	5	5	5	
Total intersection points C^n_{m+n}		10	20	35	70	126	252	3003	
Distribution E^0	$C^0_m \times C^n_n$	1	1	1	1	1	1	1	
Distribution E^1	$C^1_m \times C^{n-1}_n$	6	9	12	16	20	25	50	
Distribution E^2	$C^2_m \times C^{n-2}_n$	3	9	18	36	60	100	450	
Distribution E^3	$C^3_m \times C^{n-3}_n$	0	1	4	16	40	100	1200	
Distribution E^4	$C^4_m \times C^{n-4}_n$		0	0	1	5	25	1050	
Distribution E^5	$C^5_m \times C^{n-5}_n$				0	0	1	252	
Distribution E^6	$C^6_m \times C^{n-6}_n$						0	0	

Table 1

$$x_1/x^A_1 + x_2/x^A_2 + x_3/x^A_3 \le 1 \quad A$$

$$x_1/x^B_1 + x_2/x^B_2 + x_3/x^B_3 \le 1 \quad B$$

$$x_1/x^C_1 + x_2/x^C_2 + x_3/x^C_3 \le 1 \quad C$$

where $x_1, x_2, x_3 \ge 0$

Solution: Planes A, B, and C (taken with the sign of equality) are shown in Fig. 2. The region of feasibility Φ is the space enclosed by structural constraint planes A, B, and C and non-negative constraint coordinate planes π_{12}, π_{13}, and π_{23}. Φ is a convex hexhedron which has eight vertices.

All vertices are distinguished as follows:

1) Because three planes A, B, and C are all intercept forms which have the value ≤ 1, the origin O is a vertex of Φ.

2) Because $x^C_1 > x^B_1 > x^A_1 > 0$, $x^A_2 > x^C_2 > x^B_2 > 0$, and $x^A_3 > x^B_3 > x^C_3 > 0$, the mid-points x^A_1, x^B_2, and x^C_3 are vertices of Φ.

3) Because π^{AC}_{12}, π^{BC}_{13}, and π^{AB}_{13} do not lie in the internal space enclosed by A, B, C, π_{12}, π_{13}, and π_{23} in the first angle, none of them is the vertex of Φ. The vertices which lie in coordinate planes are π^{AB}_{12}, π^{AC}_{13}, and π^{BC}_{23}.

4) The intersection point Σ^{ABC}_{123} of three planes A, B, and C is a vertex of Φ.

Those vertices are shown in Table 2 (the vertex of Φ is denoted by V).

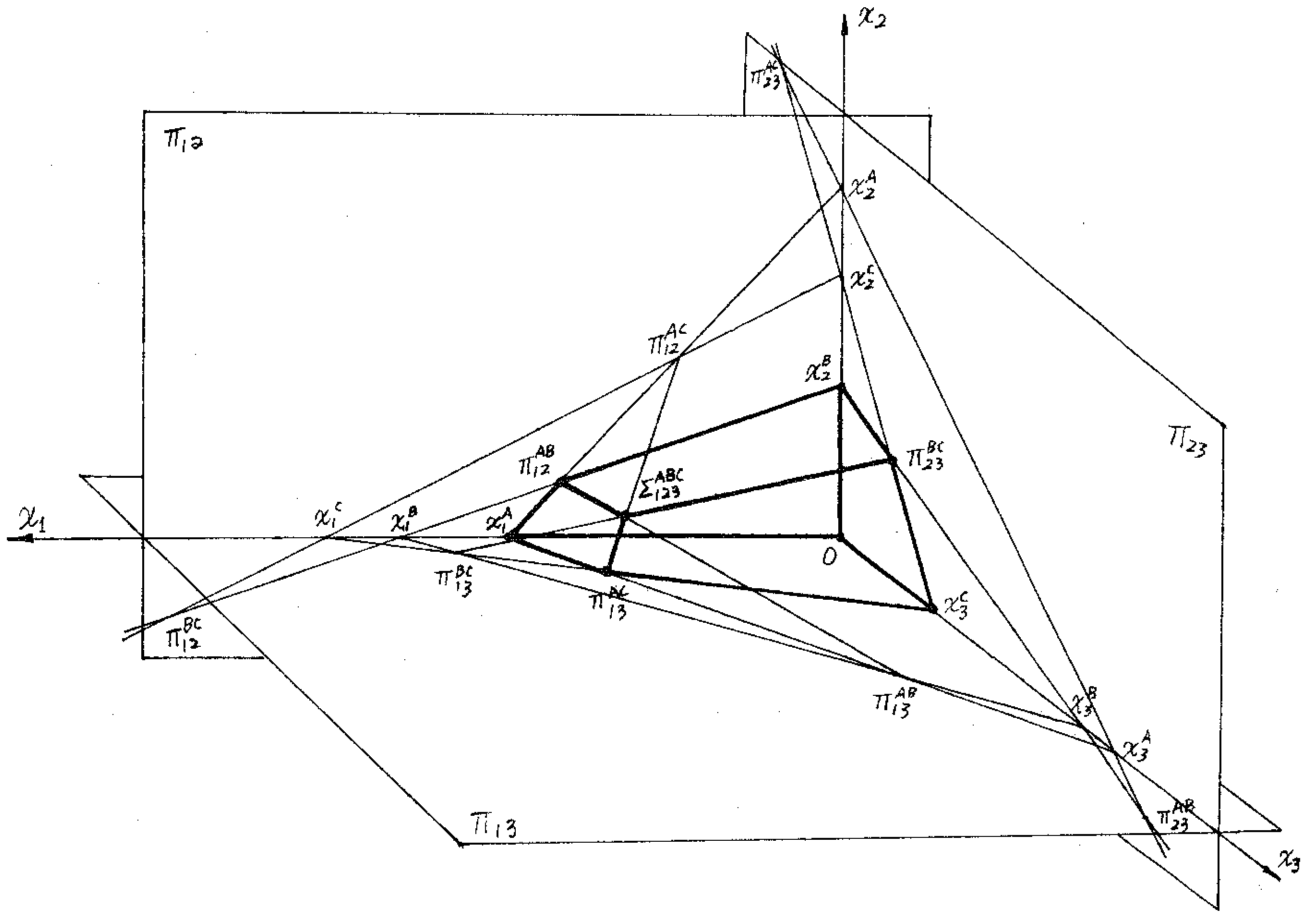

Fig. 2

Vertices	Combination of constraint inequalities: A	B	C	A∩B	A∩C	B∩C	A∩B∩C
V_o (0)							
V_1 (x_1)	$x^A{}_1$						
V_2 (x_2)		$x^B{}_2$					
V_3 (x_3)			$x^C{}_3$				
V_{12} (π_{12})				$\pi^{AB}{}_{12}$			
V_{13} (π_{13})					$\pi^{AC}{}_{13}$		
V_{23} (π_{23})						$\pi^{BC}{}_{23}$	
V_{123} (Σ_{123})							$\Sigma^{ABC}{}_{123}$

Table 2

The value of each of the above equations is ≤ 1. For the cases where the value is ≥ 1, $= 1$, and $= 0$, the explanation is analogous.

2.3 Criteria of vertices of Φ

Suppose the structural constraint hyperplanes are translated to the intercept forms:

1) The origin O

a. If each of Eqs. (2) has the value ≤ 1, the origin is V_o.

b. Among Eqs. (2), if some have the value ≤ 1 and the others have the value $= 0$, the origin is V_o.

2) The vertices on coordinate axes (taking the axis x_1, for example)

a. If each of Eqs. (2) has the value ≤ 1, the smallest positive intercept point on the axis is V_1.

b. If each of Eqs. (2) has the value ≥ 1, the largest positive intercept on the axis x_1 is V_1.

c. Among Eqs. (2), if some have the value ≤ 1 and others have the value ≥ 1, the smallest positive intercept point on the axis is denoted by V^*_1 and the largest positive intercept point is denoted by V^{**}_1. A decision is based upon the following:

(a.) If $V^*_1 < V^{**}_1$, there is no vertex on the axis x_1.

(b.) If $V^*_1 > V^{**}_1$, there are two vertices V^*_1 and V^{**}_1 on the axis x_1.

(c.) If $V^*_1 = V^{**}_1$, there is one vertex V^*_1 (or V^{**}_1) on the axis x_1.

d. If there is any equation having the value $= 1$ in Eqs. (2), only the positive intercept point of this equation on the axis x_1 may be V_1.

e. If there is any equation with the sign $= 0$ in Eqs. (2), there is no vertex on the axis x_1.

3) The vertices on coordinate planes (taking the coordinate plane π_{12} as an example)

The vertices of Φ on the coordinate planes are the basis for determining the vertices on three or more dimensional subspaces. Take $m = 5$, for example

a. Each of Eqs. (2) has the value ≤ 1 (Fig. 3).

b. Each of Eqs. (2) has the sign ≥ 1 (Fig. 4).

c. Among Eqs. (2), some have the value ≤ 1 and others have the value ≥ 1 (Fig. 5).

d. If there is any equation having the value $= 1$, the intersection point of this equation and the shadowed lines in Fig. 3 - 5 is the V_{12}.

4) Vertices on three or more dimensional subspaces

The existence of vertices of Φ on three or more dimensional subspaces depends on the distribution of vertices on the subspaces which are one dimension fewer.

The criteria are:

a. k (k-1) dimensional vertices determine one k-dimensional vertex.

b. k-2 (and only k-2) of k-1 superscripts between any two of k (k-1)-dimensional vertices are the same, superscripts denoting the hyperplanes, and subscripts in k (k-1)-dimensional vertices contain k distinct coordinate axes.

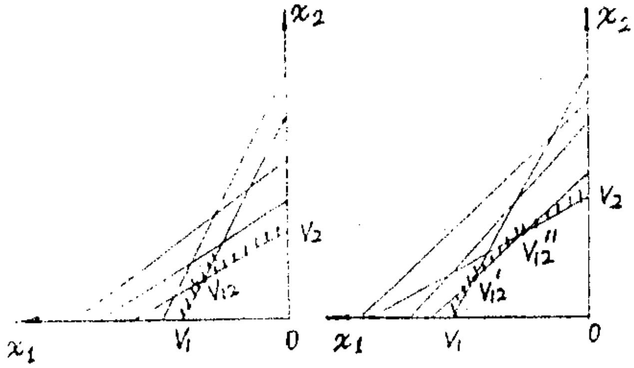

Fig. 3

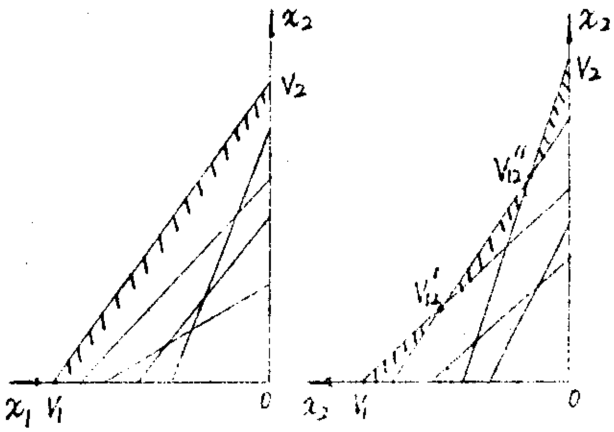

Fig. 4

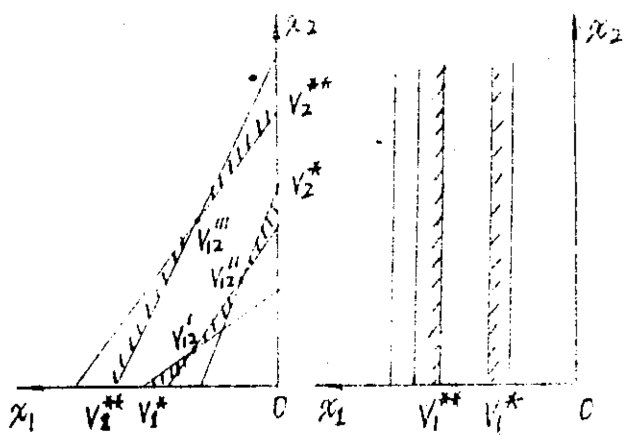

Fig. 5

Example 3

Required: Given vertices of Φ, $\pi^{AB}{}_{12}$, $\pi^{AC}{}_{13}$, $\pi^{BC}{}_{23}$, $\pi^{AD}{}_{12}$, $\pi^{BD}{}_{23}$, determine the vertices on three-dimensional subspaces.

Solution: One of two subscripts between any two of the vertices $\pi^{AB}{}_{12}$, $\pi^{AC}{}_{13}$, $\pi^{BC}{}_{23}$ are the same and subscripts of these three vertices contain three coordinate axes x_1, x_2, and x_3. Hence

$$\pi^{AB}{}_{12},\ \pi^{AC}{}_{13},\ \pi^{BC}{}_{23} \dashrightarrow \Sigma^{ABC}{}_{123}$$

(that is, $\pi^{AB}{}_{12}$, $\pi^{AC}{}_{13}$, and $\pi^{BC}{}_{23}$ determine the existence of $\Sigma^{ABC}{}_{123}$). By analogy,

$$\pi^{AB}{}_{12},\ \pi^{AD}{}_{12},\ \pi^{BD}{}_{23} \dashrightarrow \Sigma^{ABD}{}_{123}$$

Example 4

Required: Given two-dimensional vertices of Φ, $\pi^{AB}{}_{12}$, $\pi^{AC}{}_{13}$, $\pi^{AD}{}_{14}$, $\pi^{BC}{}_{23}$, $\pi^{BD}{}_{24}$, and $\pi^{CD}{}_{34}$, determine the existence of three or more dimensional vertices.

Solution: According to the analysis of Example 3,

$$\pi^{AB}{}_{12},\ \pi^{AC}{}_{12},\ \pi^{BC}{}_{23} \dashrightarrow \Sigma^{ABC}{}_{123}$$
$$\pi^{AB}{}_{12},\ \pi^{AD}{}_{14},\ \pi^{BD}{}_{24} \dashrightarrow \Sigma^{ABD}{}_{124}$$
$$\pi^{AC}{}_{13},\ \pi^{AD}{}_{14},\ \pi^{CD}{}_{34} \dashrightarrow \Sigma^{ACD}{}_{134}$$
$$\pi^{BC}{}_{23},\ \pi^{BD}{}_{24},\ \pi^{CD}{}_{34} \dashrightarrow \Sigma^{BCD}{}_{234}$$

In these four three-dimensional vertices $\Sigma^{ABC}{}_{123}$, $\Sigma^{ABD}{}_{124}$, $\Sigma^{ACD}{}_{134}$, and $\Sigma^{BCD}{}_{234}$, two of three subscripts between any two vertices are the same and their subscripts contain four axes x_1, x_2, x_3, and x_4. Hence

$$\Sigma^{ABC}_{123},\ \Sigma^{ABD}_{124},\ \Sigma^{ACD}_{134},\ \Sigma^{BCD}_{234} \rightarrow \Sigma^{ABCD}_{1234}$$

3. Solving Linear Programming Problems by the N-dimensional Geometric Model

3.1 Solution steps for the n-dimensional geometric model

1) Translate the structural constraint hyperplanes to intercept form equations.

2) Find the vertices of Φ on coordinate axes.

3) Find the vertices of Φ on coordinate planes.

4) Determine the existence of three-dimensional vertices by two-dimensional vertices and find them.

5) Determine the existence of k-dimensional vertices by (k-1)-dimensional vertices and find them.

6) Repeat the procedure in 5) until no vertex exists in some dimensional subspace.

7) Substitute coordinate values of all the vertices of Φ in the objective function, calculate it, and determine the optimal solution.

3.2 An example for solving a linear programming problem with five variables

Example 5

Required: Minimize $z = -x_1 + 2x_2 - x_3 + 3x_4 - 1.5x_5$

subject to

$$x_1/25 + x_2/45 + x_3/45 + x_4/45 + x_5/30 \le 1 \qquad A$$

$$x_1/32 + x_2/37 + x_3/40 + x_4/38 + x_5/45 \le 1 \qquad B$$

$$x_1/40 + x_2/20 + x_3/54 + x_4/28 + x_5/20 \le 1 \qquad C$$

$$x_1/50 + x_2/35 + x_3/26 + x_4/52 + x_5/35 \le 1 \qquad D$$

$$x_1/45 + x_2/50 + x_3/45 + x_4/38 + x_5/40 \le 1 \qquad E$$

where $x_1, x_2, x_3, x_4, x_5 \ge 0$

Solution: In order to solve this linear programming problem, a graphic-analytic solution may be used. This solution will describe the computer-aided graphic-analytic method.

1) Because each of the structural constraint equations has the value ≤ 1, the origin O is a vertex of Φ.

2) After comparing the intersections on axes, x^A_1, x^C_2, x^D_3, x^C_4, and x^C_5 are vertices of Φ on axes.

3) Determine the vertices of Φ on coordinate planes with an analytic method. They are π^{AC}_{12}, π^{AD}_{13}, π^{AC}_{14}, π^{AC}_{15}, π^{CD}_{23}, π^{CD}_{34}, and π^{CD}_{35} (Table 3).

4) From the criteria of Sec. 2.3,

$$\pi^{AC}_{12},\ \pi^{AD}_{13},\ \pi^{CD}_{23} \rightarrow \Sigma^{ACD}_{123}$$
$$\pi^{AD}_{13},\ \pi^{AC}_{14},\ \pi^{CD}_{34} \rightarrow \Sigma^{ACD}_{134}$$
$$\pi^{AD}_{13},\ \pi^{AC}_{15},\ \pi^{CD}_{35} \rightarrow \Sigma^{ACD}_{135}$$

Vertices	Coordinate Values					Objective Values
	x_1	x_2	x_3	x_4	x_5	
0	0.000	0.000	0.000	0.000	0.000	0.000
$x^A{}_1$	25.000	0.000	0.000	0.000	0.000	-25.000
$x^C{}_2$	0.000	20.000	0.000	0.000	0.000	50.000
$x^D{}_3$	0.000	0.000	26.000	0.000	0.000	-26.000
$x^C{}_4$	0.000	0.000	0.000	28.000	0.000	84.000
$x^C{}_5$	0.000	0.000	0.000	0.000	20.000	-30.000
$\pi^{AC}{}_{12}$	19.231	10.385	0.000	0.000	0.000	1.593
$\pi^{AD}{}_{13}$	14.844	0.000	18.281	0.000	0.000	-33.125
$\pi^{AC}{}_{14}$	15.455	0.000	0.000	17.182	0.000	36.091
$\pi^{AC}{}_{15}$	14.286	0.000	0.000	0.000	12.857	-33.572
$\pi^{CD}{}_{23}$	0.000	14.307	15.872	0.000	0.000	13.242
$\pi^{CD}{}_{34}$	0.000	0.000	16.200	19.600	0.000	42.600
$\pi^{CD}{}_{35}$	0.000	0.000	15.372	0.000	14.307	-36.833
$\Sigma^{ACD}{}_{123}$	13.084	8.758	12.758	0.000	0.000	-8.258
$\Sigma^{ACD}{}_{134}$	9.298	0.000	14.066	14.198	0.000	19.230
$\Sigma^{ACD}{}_{135}$ Optimal Solution	8.518	0.000	13.626	0.000	10.694	-38.185 Optimal Value

Table 3

5) Only four three-dimensional vertices can generate one four-dimensional vertex. Because there are only three three-dimensional vertices in this example, there is no vertex on four or more dimensional subspaces.

For finding three vertices $\Sigma^{ACD}{}_{123}$, $\Sigma^{ACD}{}_{134}$, and $\Sigma^{ACD}{}_{135}$, use $\Sigma^{ACD}{}_{123}$ as an example.

$\Sigma^{ACD}{}_{123}$ is the common point of three edge lines $\pi^{AC}{}_{12}\Sigma^{ACD}{}_{123}$, $\pi^{AD}{}_{13}\Sigma^{ACD}{}_{123}$, and $\pi^{CD}{}_{23}\Sigma^{ACD}{}_{123}$. The above three edge lines are three segments $\pi^{AC}{}_{12}\pi^{AC}{}_{13}$, $\pi^{AD}{}_{13}\pi^{AD}{}_{12}$, and $\pi^{CD}{}_{23}\pi^{CD}{}_{13}$ that lie in Σ_{123}. Because any two of these three segments intersect, choose any two (such as $\pi^{AC}{}_{12}\pi^{AC}{}_{13}$ and $\pi^{AD}{}_{13}\pi^{AD}{}_{12}$) to find the intersection point.

Write the coordinates of the end points of those segments:

$\pi^{AC}{}_{12}\pi^{AC}{}_{13}$: $\pi^{AC}{}_{12}$(19.231,

10.385,0,0,0)
$\pi^{AC}{}_{13}$(-20,0,81,0,0)

$\pi^{AD}{}_{13}\pi^{AD}{}_{12}$: $\pi^{AD}{}_{13}$(14.844, 0,18.281,0,0)
$\pi^{AD}{}_{12}$(9.091,28.636, 0,0,0)

In order to simplify calculations, use parameter equations to represent the segments. Solve the parameter equations to determine that the parameters are 0.1516 and 0.3058. Substitute them into the parameter equations to find the point of intersection

$\Sigma^{ACD}{}_{123}$ (13.084, 8.758, 12.690, 0, 0)

By analogy

$\Sigma^{ACD}{}_{134}$ (9.298, 0, 14.060, 14.198, 0)
$\Sigma^{ACD}{}_{135}$ (8.518, 0, 13.626, 0, 10.694)

With this step, all vertices of Φ have been found. Their coordinate values are in Table 3.

6) Substitute the coordinate values of the obtained vertices in the objective function and determine the optimal solution and the optimal values, also shown in Table 3.

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Acknowledgement

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Formulae for Numerically Determining Line of Intersection and Dihedral Angle Between Two Planes

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In civil engineering it is often necessary to find the line of intersection of two plane segments which are defined by their strike and dip angles. When this problem occurs, a general method for numerically finding the bearing and slope angles for the line of intersection is very helpful. The formulae for determining these two angles is developed based on the properties of trigonometry. By substituting the strike and dip angles of two given plane segments directly into the formulae, extremely accurate values can be calculated for the bearing and slope angles of the line of intersection. The formula for numerically determining the dihedral angle between two plane segments is also developed.

Existing Method

A common method for finding the line of intersection between two plane segments, which is described in most descriptive geometry textbooks, employs auxiliary projection methods using the following steps:

1. Locate and specify the plane segments as edge views using given strike and dip angles as shown in Fig. 1. Since in the top view the given strike lines 1-2 and 3-4 are horizontal and thereby true-length, the point view of each strike line is found by primary auxiliary views, using a common reference plane. The edge views of the plane segments can next be established by constructing the dip angle with the folding line H-1 through the point views. The low side is the side of the dip direction.

2. Construct a horizontal reference plane H'-F' at a convenient location, say a distance G from the H-F plane, in the front view as shown in Fig. 2. This plane is then projected to both auxiliary views in which the plane is located the same distance G from the folding line H-1. The H-1 and H'-1' planes cut through each plane segment in the auxiliary views to locate points M and N, respectively, on each plane segment.

3. Complete the line of intersection by finding the points M and N in the top and front Views. Since the point M in the top view is found by projecting from each auxiliary view, the points N on the H'-1' plane are also projected to their intersection in

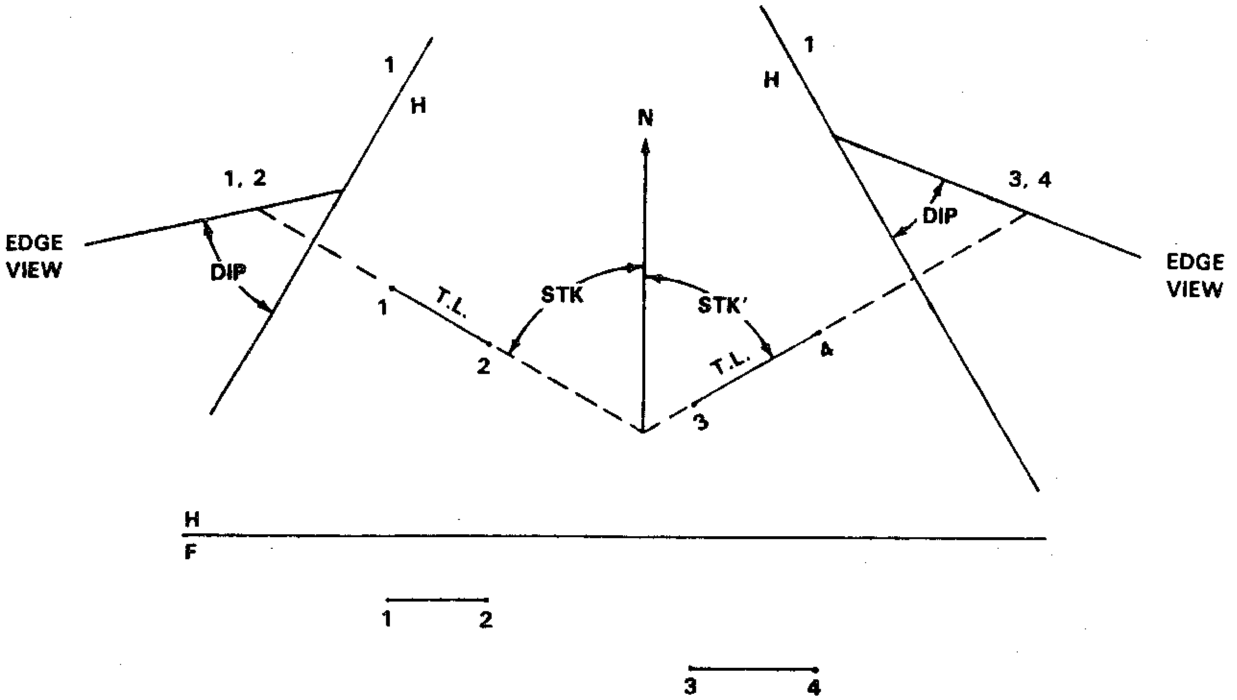

Fig. 1 Construction of edge views of given planes for strike and dip angles

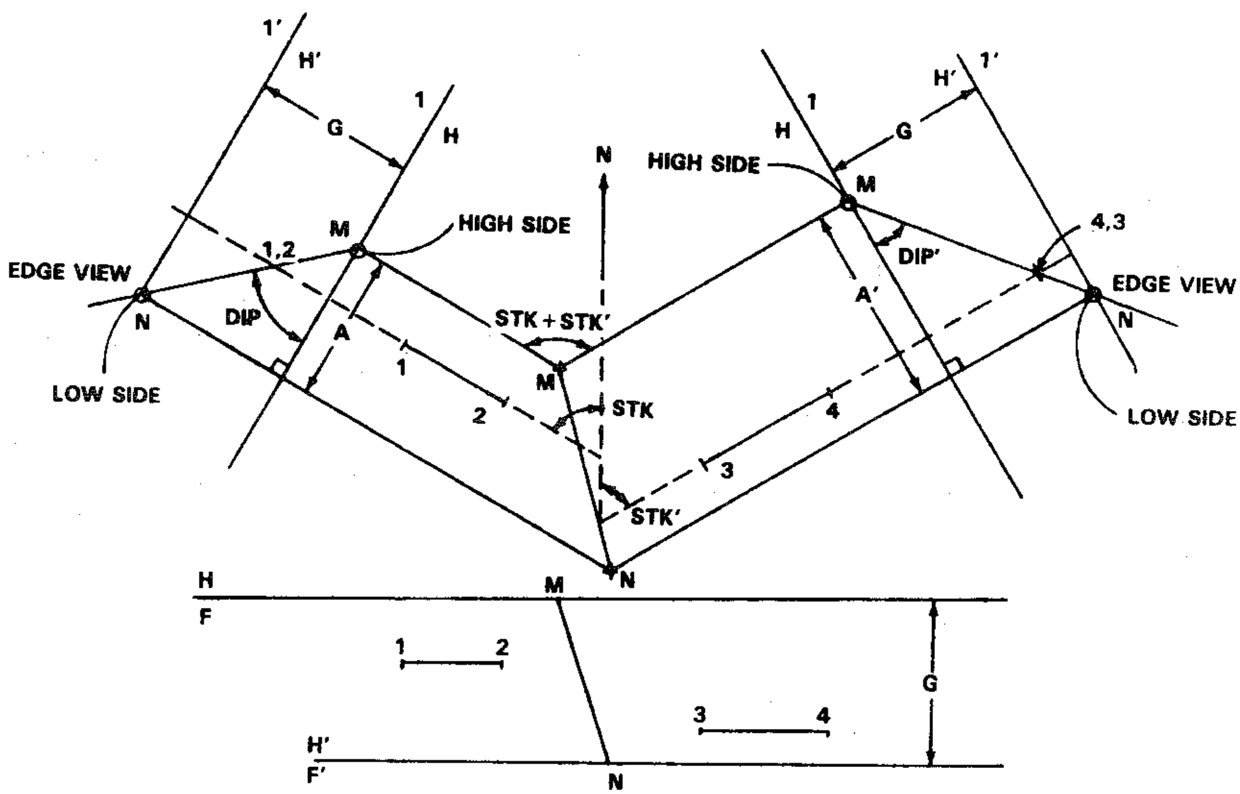

Fig. 2 Graphical solution to find line of intersection by auxiliary projection method

the top view at point N. Points M and N are then projected to their respective planes in the front view. Line M-N is the line of intersection between the two plane segments.

This method has to be accomplished by means of graphics and the accuracy of its results strictly depends on the scale of the drawing and the skill of the draftsman.

New Method

The formulae proposed for determining the bearing angle, BRG, and the slope angle, SLP, for the line of intersection using the strike and dip angles of two given plane segments are:

$$BRG = STK' + SUB + 90°$$

$$SLP = \tan^{-1}[\tan(DIP')\cos(SUB)]$$

where

$$SUB = \tan^{-1}\{\tan(DIP')/[\tan(DIP)\cos(MAG)] - \tan(MAG)\}$$

and

$$MAG = STK + STK' - 90°$$

BRG is the azimuth bearing angle that is measured clockwise from the North-axis. SLP is the slope angle of the line of intersection. STK and DIP represent the strike and dip angles of one plane segment as STK' and DIP' represent the strike and dip angles of the other plane segment. However, STK must be the angle that the strike line in the North-West (or South-East) direction makes with the North-axis and STK' must be the angle that the strike line in the North-East (or South-West) direction makes with the North axis.

The proposed formula for determining the dihedral angle between the two given plane segments is:

$$DIH = 180° - \tan^{-1}[\cos(MAG + SUB)\cos(90° - SLP)/\sin(MAG + SUB)] - \tan^{-1}[\cos(90° - SUB)\cos(90° - SLP)/\sin(90° - SUB)]$$

Development of Formulae

1. Bearing Angle for the Line of Intersection

The two right triangles in the auxiliary views of Fig. 2 give the following relations:

$$A = G / \tan(DIP) \quad (1)$$

and

$$A' = G / \tan(DIP') \quad (2)$$

where dip angle DIP and DIP' are given and length G can be eliminated (through cancelation) as the development is carried further. The two right triangles to be observed next are M-9-11 and M-10-12, which are shaded as shown in Fig. 3. The dimensions B and C' can then be expressed as:

$$B = A / \cos(MAG) \quad (3)$$

and

$$C' = A' \tan(MAG) \quad (4)$$

where angle MAG is 90° smaller than the sum of the two strike angles. Thus,

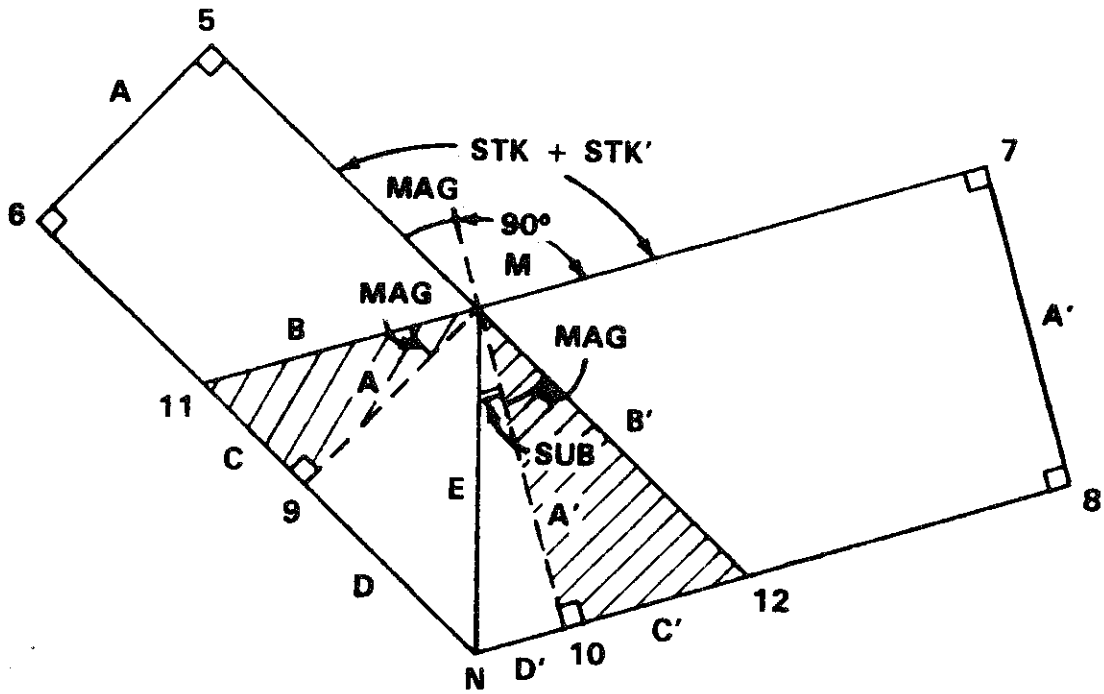

Fig. 3 Dimensions required for formula development of bearing and slope angles for line of intersection

$$MAG = STK + STK' - 90°$$

The parallelogram M-11-N-12 in Fig. 3 (which is developed from Fig. 2) must be used next. As one side B of the parallelogram is equal to the opposite side (C' + D'), the dimension D' can be defined as follows:

$$D' = B - C' \tag{5}$$

Angle SUB in the right triangle M-N-10 in the same figure can be obtained by

$$SUB = \tan^{-1}(D'/A') \tag{6}$$

By substituting Eqs. (1), (2), (3), (4), and (5) into Eq. (6), SUB may be defined as

$$SUB = \tan^{-1}\{\tan(DIP')/[\tan(DIP)\cos(MAG)] - \tan(MAG)\}$$

However, the line of intersection (line M-N) is at an angle which is the sum of angle SUB, a right angle , and angle STK', from the North-axis as shown in Fig. 4. Thus,

$$BRG = STK' + SUB + 90°$$

2. Slope Angle for the Line of Intersection

Since the slope of any given line is defined as the ratio of the rise of the line to the run, the slope angle for the line of intersection SLP can be expressed as

$$SLP = \tan^{-1}(G/E) \tag{7}$$

where G represents the rise (from Fig. 3) and E represents the run (from Fig. 2) for the line of intersection. The dimension E is

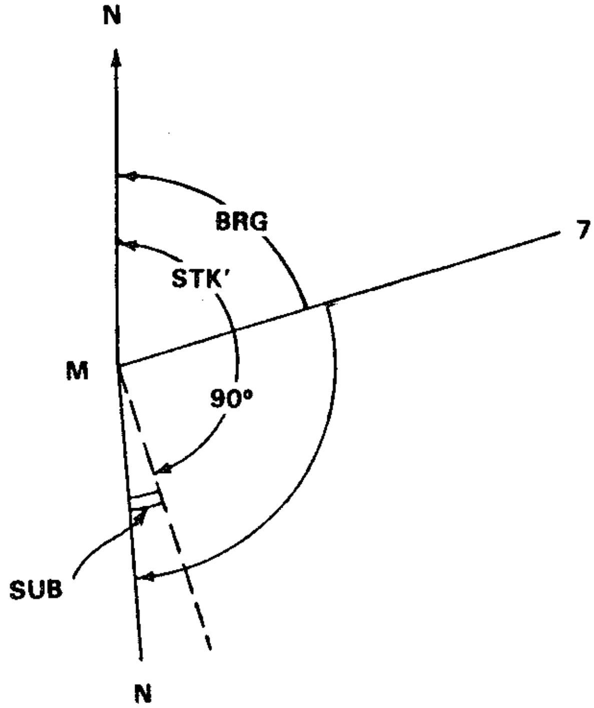

Fig. 4 Azimuth bearing angle, BRG, of line of intersection, M-N

unknown at this point. However, from the right triangle M-N-10 in Fig. 3, length E can be obtained as

$$E = A'/\cos(SUB) \quad (8)$$

Then, sudstituting Eq. (2) into Eq. (8), the same equation can be rewritten as

$$E = G/[\tan(DIP')\cos(SUB)] \quad (9)$$

Substituting Eq. (9) into Eq. (7), the same equation can be developed into the final form

$$SLP = \tan^{-1}[\tan(DIP')\cos(SUB)]$$

3. Dihedral Angle of Two Planes

Figure 5 was constructed graphically by following the principles of projection (the front view is not shown). Two triangular planes M-N-5 and M-N-7 are established and then employed to find the dihedral angle. The true dihedral angle is found in the secondary auxiliary view showing the line of intersection as a point and can be determined numerically with

$$DIH = 180° - GAM - LAM \quad (10)$$

where angles GAM and LAM remain unknown at this point. If these two angles are to be determined, the following investigation must be carried out.

First, in the top view in Fig. 5, find the two shaded right triangles which would give:

$$R = X \sin(ALF) \quad (11)$$
$$O = X \cos(ALF) \quad (12)$$
$$S = Y \sin(BET) \quad (13)$$
$$P = Y \cos(BET) \quad (14)$$

In the first auxiliary view in the same figure, the following relations stand according to the two overlapping right triangles sharing the same angle (90° - SLP):

$$U = O \cos(90° - SLP) \quad (15)$$
$$V = P \cos(90° - SLP) \quad (16)$$

Substituting Eq. (12) into (15) and Eq. (14) into (16), the same two equations can be rewritten as:

$$U = X \cos(ALF)\cos(90° - SLP) \quad (17)$$
$$V = Y \cos(BET)\cos(90° - SLP) \quad (18)$$

From the right triangles in the secondary auxiliary view of Fig. 5,

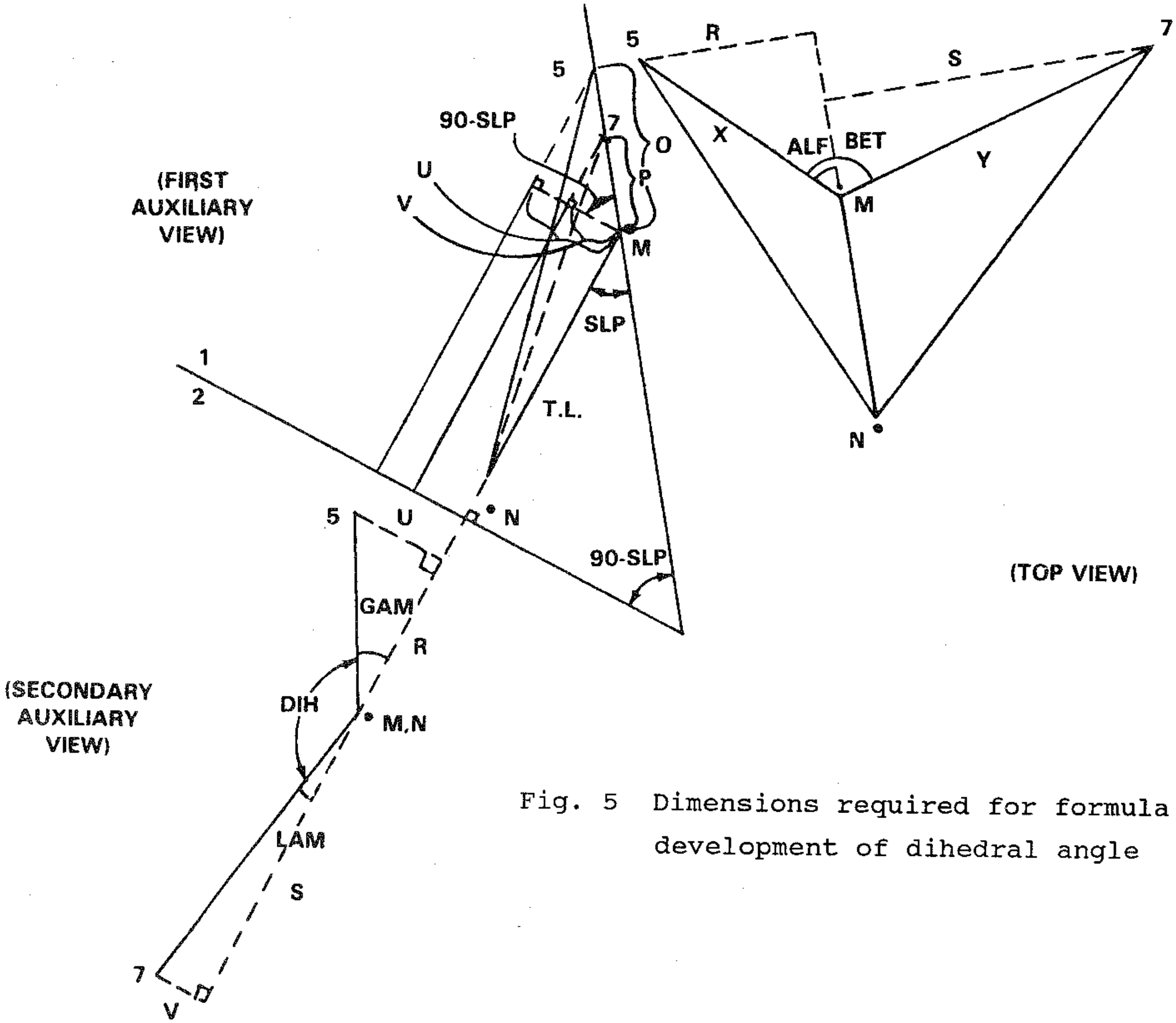

Fig. 5 Dimensions required for formula development of dihedral angle

$$GAM = \tan^{-1}(U/R)$$
$$LAM = \tan^{-1}(V/S)$$

Substituting Eqs. (11) and (17) into the equation for GAM and substituting Eqs. (13) and (18) into the equation for LAM yields

$$GAM = \tan^{-1}[\cos(ALF)\cos(90° - SLP)/\sin(ALF)] \quad (19)$$

$$LAM = \tan^{-1}[\cos(BET)\cos(90° - SLP)/\sin(BET)] \quad (20)$$

Finally, substituting Eqs. (19) and (20) into (10), the dihedral angle can then be expressed as

$$DIH = 180° - \tan^{-1}[\cos(ALF)\cos(90°-SLP)/\sin(ALF)] - \tan^{-1}[\cos(BET)\cos(90° - SLP)/\sin(BET)]$$

where angles ALF and BET, which are related to angles STK + STK' and SUB as shown in Fig. 6, can be written as

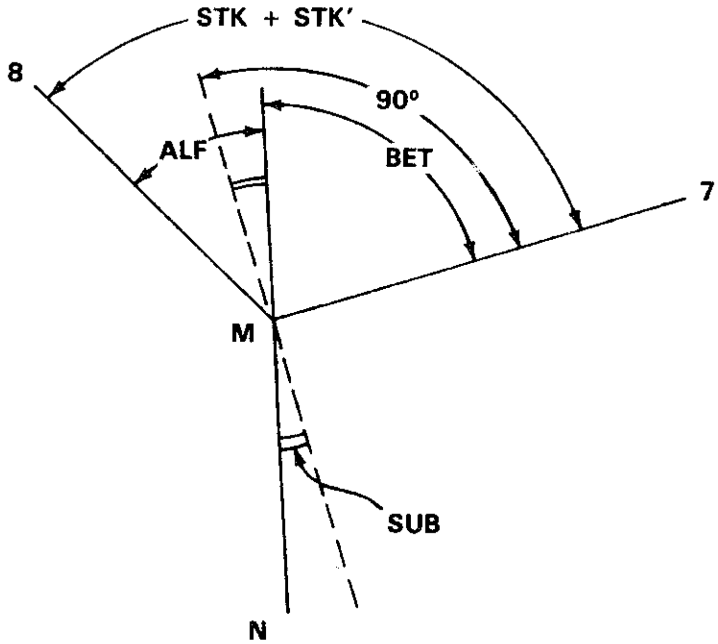

Fig. 6 Angles ALF and BET for determining dihedral angle

$$BET = 90° - SUB$$
$$ALF = STK + STK' - BET$$
$$= STK + STK - 90° + SUB$$
$$= MAG + SUB$$

Use of Formulae

There is a simple procedure to follow if employing the formulae of the "New Method":

1. Identify angles STK, STK', DIP, and DIP'

2. Calculate angle MAG.

3. Calculate angle SUB.

4. Determine the azimuth bearing angle for the line of intersection, BRG.

	PLANE 1		PLANE 2	
PROB	STRIKE	DIP	STRIKE	DIP
A	N60°W	47°SW	N60°E	40°SE
B	N74°W	45°NE	N45°E	40°NW
C	N82°E	22°SE	N42°W	39°SW

Table 1. List of example problems.

PROB	STK	STK'	DIP	DIP'	MAG	SUB	BRG	SLP	DIH
A	60	60	47	40	30	18.065	168.065	38.581	139.243
B	74	45	45	40	29	22.052	157.052	37.873	139.644
C	42	82	39	22	34	-4.158	167.842	21.948	148.472

Table 2. List of tabulated values at each step.

5. Determine the slope angle for the line of intersection, SLP.

6. Finally, estimate the dihedral angle between two plane segments, DIH.

Three example problems, illustrated in Table 1 are provided to demonstrate the use of the formulae. The tabulated values at each step following the above procedure are listed in Table 2. The calculation of these problems was carried out directly on a simple scientific calculator case by case. However, if there were a large volume of the problems to be solved, the proposed formulae may also be written in a computer language, such as BASIC or FORTRAN. The number of decimal places for BRG, SLP,and DIH can be as significant as necessary and limited only by the computational device.

Spirograph

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Specific characteristics of various spirographs are described. It is shown that (1) the curve generated by the locus of a point on or within a circle of radius B rolling inside a fixed circle of radius A creates the same figure as the locus of a similarly located point on or within a circle of radius A-B rolling inside a fixed circle of radius A, (2) N-leaved rose curves are spirographs, implying a transformation exists between the two curve forms, and (3) a transformation exists between the hypocycloid and the epicycloid.

Introduction

The hypocycloid and epicycloid are called spirographs. Special cases of the spirograph include deltoids, asteroids, cardioids, and nephroids.

The Hypocycloid

The spirograph is described by the following system of parametric equations:

$$X = (A - B)\cos\Theta + D\cos\alpha$$
$$Y = (A - B)\sin\Theta - D\sin\alpha \qquad (1)$$

where A is the radius of a circle in which another circle of radius B rolls and D is defined as the eccentricity. Θ is defined as the angular displacement while α is related to A, B, and Θ by the equation

$$\alpha = (A - B)\Theta/B$$

as shown in Fig. 1. Figure 2 illustrates the pattern which is drawn with A = 9, B = 2, and D = 3.5.

If integers A and B are mutually prime, A is the total number of blades and B - 1 is the number of blades which is skipped when the pen is plotting the spirograph. B is also the least number of rotations (n) of the moving circle (radius B) around the fixed circle (radius A). Therefore, n = B. The pen returns to its original position when $\Theta = 2\pi B$ (Fig. 3)

If integers A and B are not mutually prime, the highest common factor (H) of B and A must be calculated. Then terms A and B are replaced by A/H and B/H giving n = B/H.

Special Case I

The generalized hypocycloid is formed when D = B in Eqs. (1). Figures 3 and 4 show the various shapes.

For the third graph in Fig. 4

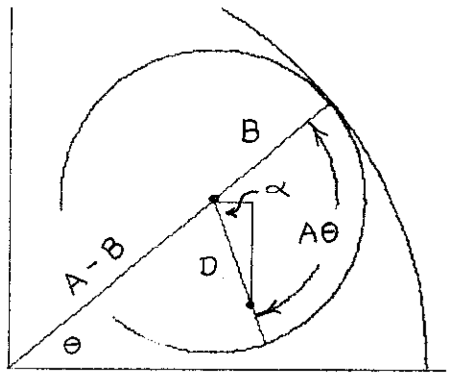


Fig. 1 Geometry defined

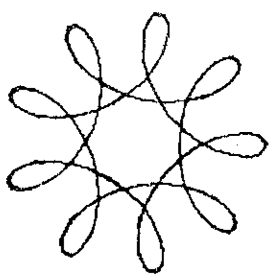


Fig. 2 Hypocycloid: A=9, B=2, D=3.5

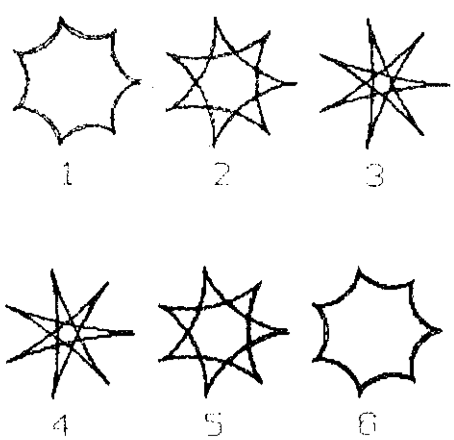


Fig. 3 Hypocycloids: A=7, B=D=

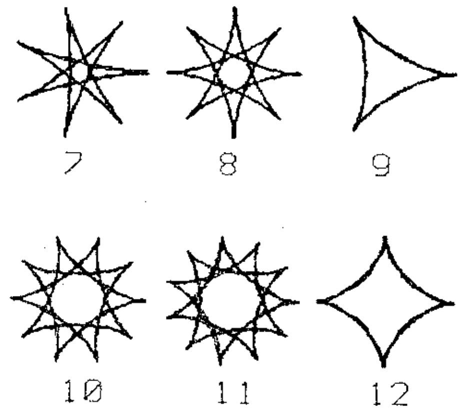


Fig. 4 Hypocycloids: B=D=3, A=

in which A = 9 and B = D = 3, Eqs. (1) become

$$X = H[2\cos\theta + \cos(2\theta)]$$
$$Y = H[2\sin\theta - \sin(2\theta)]$$

where H = 3. These represent the equations of the deltoid which is a hypocycloid of three cusps formed when A = 3B.

For the sixth graph in Fig. 4 in which A = 12 and B = D = 3, Eqs. (1) become

$$X = a\cos^3\theta$$
$$Y = a\sin^3\theta$$

or

$$X^{2/3} + Y^{2/3} = a^{2/3}$$

where a = 4H, with H = 3 in this example. These equations represent the equation of the asteroid. It is a hypocycloid of four cusps, formed when A = 4B.

Special Case II

When D = A - B in Eqs. (1), the parametric equations of the hypocycloid become

$$X = (A - B)(\cos\Theta + \cos\alpha)$$
$$Y = (A - B)(\sin\Theta - \sin\alpha) \quad (2)$$

Rewritten, Eqs. (2) become

$$X = 2(A - B)\cos[(\Theta + \alpha)/2]\cdot\cos[(\Theta - \alpha)/2]$$
$$Y = 2(A - B)\cos[(\Theta + \alpha)/2]\cdot\sin[(\Theta - \alpha)/2] \quad (3)$$

In Eqs. (3), when $\cos[(\Theta + \alpha)/2] = 0$, both X and Y are equal to zero. Thus

$$(\Theta + \alpha)/2 = (2n - 1)\pi/2$$

or, using $\alpha = (A - B)\Theta/B$

$$\Theta = B(2n - 1)\pi/A$$

where n = 1, 2, 3, ..., yielding a graph which is a hypocycloid passing through the center of the pattern (Fig. 5).

For values of B from 2 to 11, each B is prime to A = 13. Therefore there are 13 blades in each pattern. By inspection, the closer the value of B is to the value of A/2, the thinner the blade becomes.

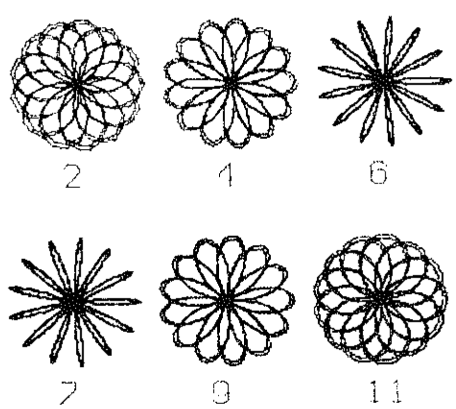


Fig. 5 Hypocycloids: A=13, D = A-B, B=

The Size of the Pattern

If $\Theta = 0$, $\cos\Theta = 1$ and $\alpha = 0$. Therefore $\cos\alpha = 1$. Then, from the first of Eqs. (1)

$$X = A - B + D \quad (4)$$

X is the length of the blade. The examples of Fig. 5, where X = 22, 18, 14, 12, 8, and 4, respectively, have different blade lengths, but the six patterns are the same size because they were scaled by computer.

Similar Figures

In Figs. 3 and 5 the substitution of A - B for B yields the same figure as described by parametric Eqs. (1), but the direction of the pen motion in plotting the figures is reversed, i.e., clockwise instead of counterclockwise. The proof of this statement follows:

Let B1 = A - B. Therefore B = A - B1. From Eqs. (1)

$$\alpha = (A - B)\Theta/B$$
$$= (B1)\Theta/(A - B1)$$

Therefore

$$\Theta = (A - B1)\alpha/B1$$

A new form of parametric equations of the hypocycloid may be established.

$$X1 = (A - B1)\cos\Theta + D1\cos\alpha$$
$$Y1 = (A - B1)\sin\Theta - D1\sin\alpha \quad (5)$$

which may be rewritten as

$$X1 = D1\cos\alpha + B\cos\Theta$$

$$Y1 = -(D1\sin\alpha - B\sin\Theta) \quad (6)$$

where $\Theta = (A - B1)\alpha/B1$ and D1 represents an unknown eccentricity. The parameter Θ is replaced by α, but the new parametric equations are still equations of a hypocycloid. Furthermore, Y1 is negative which indicates that the direction of pen motion when plotting is reversed.

Equations (6) and Eqs. (1) yield similar figures under certain conditions. The relationship among B, B1, D, and D1 must be considered.

Compare the first of Eqs. (1)

$$X = (A - B)\cos\Theta + D\cos\alpha$$
$$= B1(\cos\Theta + D\cos\alpha/B1)$$

since B1 = A - B with the first of Eqs. (6)

$$X1 = D1(\cos\alpha + B\cos\Theta/D1)$$

The condition of similarity is

$$D/B1 = B/D1$$

or

$$D1 = B(A - B)/D \quad (7)$$

Special Case I

In Eqs. (1) the generalized hypocycloid is formed when D = B. Substituting B for D in Eqs. (7) yields D1 = A - B or D1 = B1. And substituting for D1 in Eqs. (5), a generalized hypocycloid also results. Moreover, the lengths of the blades may be calculated using Eq. (4) to obtain

$$X = X1 = A$$

Thus, the two figures are not only similar but also congruent. Another example is shown in Fig. 6 in which (a) A = 3, B = 1, D = 1; (b) A = 3, B1 = A - B = 2, D1 = 2.

Special Case II

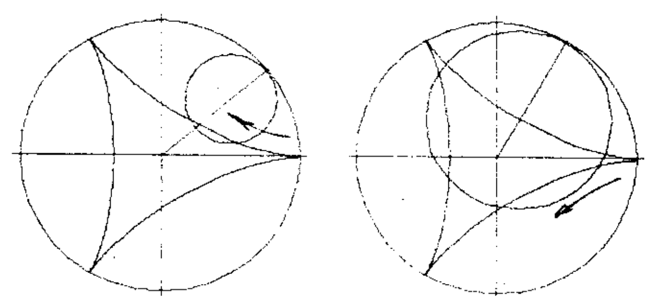
In Eqs. (1), when D = A - B = B1, the substitution of Eq. (7) yields D1 = B = A - B1. Substituting for D1 in Eqs. (5) yields a hypocycloid passing through the center of the pattern also. Calculating the lengths of the blades using Eq. (4) yields

$$X = A - B + D$$
$$= 2(A - B)$$
$$= 2(B1)$$

and

$$X1 = A - B1 + D1$$
$$= 2(A - B1)$$
$$= 2B$$

Thus, X = X1. The two resulting graphs are similar figures and the direction of the plotting the figure is reversed because Y1 is negative, as illustrated in Fig. 5.



(a) (b)

Fig. 6 Hypocycloid generation

Special Case III

An interesting problem occurs when D1 = D giving

$$D \text{ or } D1 = \sqrt{B\ B1}$$

From A = 3 to A = 8, the values of B, B1, D, and D1 are given in Table 1. Similar figures result when A = 3, B = 1, and D = $\sqrt{2}$; and when A = 3, B1 = 2, and D1 = $\sqrt{2}$, as shown in Fig. 7.

For different values of D, as illustrated in Fig. 8, the hypocycloid may be classified as follows:

(a) If D < B, the trace is close to the boundary.

(b) If D = B, the curve is a generalized hypocycloid.

(c) If B < D < A - B, the curve loops to form a hollow flower.

(d) If D = A - B, the curve is a hypocycloid passing through the center of the pattern.

(e) If D > A - B, the curve loops over the center.

A	B	B1	D or D1
3	1	2	$\sqrt{2}$
4	1	3	$\sqrt{3}$
5	1	4	2
5	2	3	$\sqrt{6}$
6	1	5	$\sqrt{5}$
7	1	6	$\sqrt{6}$
7	2	5	$\sqrt{10}$
7	3	4	$2\sqrt{3}$
8	1	7	$\sqrt{7}$

Table 1

Rose Curves

Equations of the form

$$r = a\cos(n\beta)$$
$$r = a\sin(n\beta)$$

have graphs which are called rose curves. These may be transformed into hypocycloids passing through the center of the design and vice-versa. To prove this, let $\beta = (\Theta - \alpha)/2$, $n\beta = (\Theta + \alpha)/2$, and

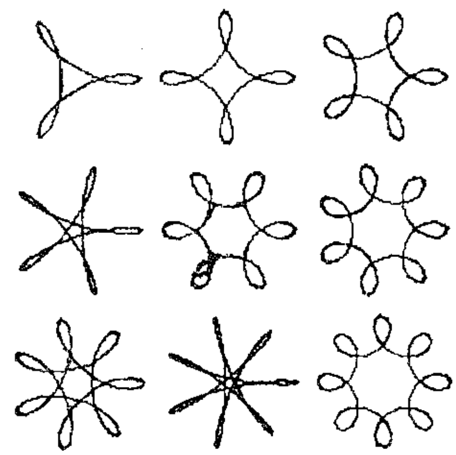


Fig. 7 Hypocycloids

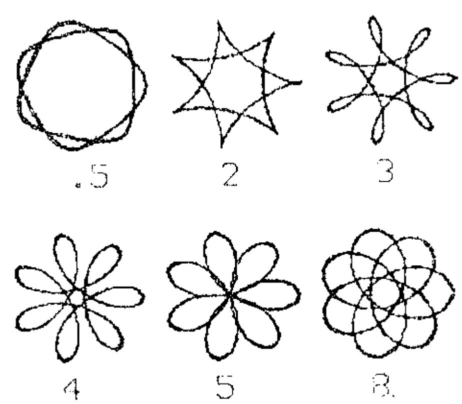


Fig. 8 Hypocycloids: A=7, B=2, D=

$a = 2(A - B)$. Eqs. (3) may then be written as

$$X = a\cos(n\beta)\cos(\beta)$$
$$Y = a\cos(n\beta)\sin(\beta)$$

The corresponding equation in polar coordinates is

$$r = a\cos(n\beta)$$

This is a standard form of the rose curve. Since

$$\alpha = (A - B)\Theta/B$$

substituting for α gives

$$\beta = (\Theta - \alpha)/2 = (2B - A)\Theta/2B$$

or

$$\Theta = 2B\beta/(2B - A) \qquad (8)$$

and

$$n\beta = (\Theta + \alpha)/2 = A\Theta/2B = A\beta/(2B - A)$$

Therefore

$$n = A/(2B - A) \qquad (9)$$

This represents the relationship between the rose cure and the hypocycloid passing through the pattern center.

A factor C is required to construct the curves. Let $\beta = 2\pi C$ when the pen returns to its original position. Since $\Theta = 2\pi B$ in the equations of a hypocycloid, substituting in Eq. (8) yields

$$C = |B - A/2| \qquad (10)$$

Example: Transform the hypocycloids in Fig. 5 into rose curves in which A = 13 and B = 2, 4, 6, 7, 9, 11.
Solution: Substitute the values for A and B into Eqs. (9) and (10) yields

$$n = -13/9,\ -13/5,\ -13,\ 13,\ 13/5,\ 13/9$$

and

$$C = 4.5,\ 2.5,\ 0.5,\ 0.5,\ 2.5,\ 4.5$$

With $a = 2(A - B) = 22, 18, 14, 12, 8, 4$, the equations of the rose curve are as follows:

$$r = 22\cos(-13\beta/9) = 22\cos(13\beta/9)$$

$$r = 18\cos(13\beta/5)$$

etc.

Fig. 9 illustrates patterns which are created by equations $r = a\cos(2\beta)$ and $r = a\cos(5\beta/3)$.

The rose curve may be transformed into a hypocycloid. From Eq. (9)

$$B/A = (1 + n)/2n$$

n is not only a positive integer, but also may be negative, and may be a decimal fraction (not an ir-

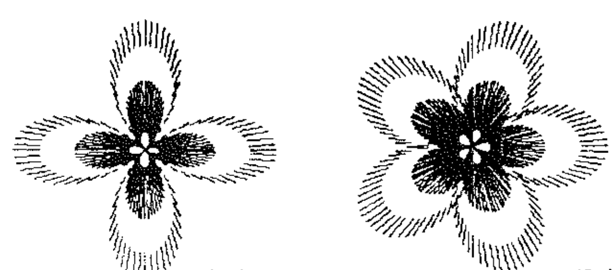

(a) (b)

Fig. 9 Rose Curves

rational number). In order to avoid a decimal point in computer calculations, let n = u/d where u and d are integers. Hence

$$B/A = (u + d)/2u \qquad (11)$$

Let A1 = 2u and B1 = u + d to find the highest common factor (H) of A and B. Then A = A1/H and B = B1/H.

Example: Transform the rose curves in Fig. 9 into hypocycloids.

(a) For n = 2, u = 2, and d = 1, A1 = 2u = 4, B1 = u + d = 3, and H = 1. Therefore, A = 4, B = 3, $C = |B - A/2| = 1$. This denotes the pattern which has four blades.

(b) For n = 5/3, A1 = 10, B1 = 8, and H = 2. Therefore A = A1/H = 5, B = B1/H = 4, and C = 1.5. This pattern has five blades.

Substituting the values of A and B into Eqs. (2) yields the parametric equations of the hypocycloid passing through the center of the pattern.

If the rose curves $r = a \sin(n\beta)$ are transformed into hypocycloids, first employ $r = a \cos(n\beta)$ to determine A and B by Eq. (11). Then substitute the values of A and B into Eqs. (2) and rotate the axes through the angle $-\pi/2n$. Because $\sin \Theta = \cos(\pi/2 - \Theta) = \cos(\Theta - \pi/2)$, it denotes that the curves are identical except the sine function is rotated by an amount $-\pi/2$ such that $n\beta = -\pi/2$. Therefore $\beta = -\pi/2n$.

The Epicycloid

The parametric equations of an epicycloid are

$$X = (A + B)\cos\Theta - D\cos\alpha$$
$$Y = (A + B)\sin\Theta - D\sin\alpha \qquad (12)$$

where $\alpha = (A + B)\Theta/B$

Case I A > B

According to different values of D in Fig. 10, a classification of the epicycloid is as follows:

(a) If D < B the trace is close to the boundary.
(b) If D = B it is a generalized epicycloid.
(c) If B < D < A + B the curve loops toward the center.
(d) If D = A + B the curve is a epicycloid passing through the center of the pattern.
(e) If D > A + B the curve loops over the center.

Fig. 11 shows generalized epicycloids in which the first pattern is a nephroid, an epicycloid of two cusps formed when A = 2B.

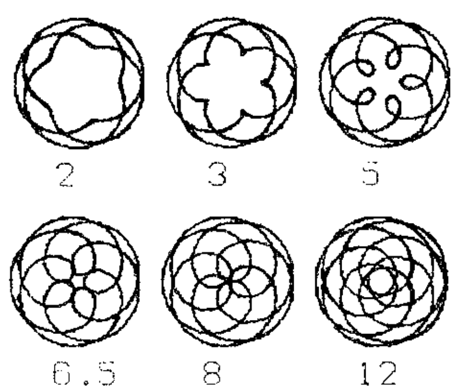


Fig. 10 Epicycloids: A=5, B=3, D=

In the third figure of Fig. 11 and in Fig. 10, the blades of all patterns overlap since B is greater than 1.

Case II $A \leq B$

Because epicycloids are formed when a moving circle rolls around the outside of a fixed circle, the radius B of the moving circle may be equal to or greater than the radius of the fixed circle. Fig. 12 illustrates the parameters.

The first curve in Fig. 12 is a cardioid, a curve of one cusp and is formed when A = B. Various values of D give the cardioids of Fig. 13.

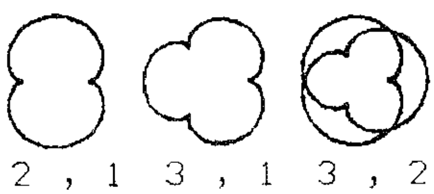


Fig. 11 Epicycloids: A=, B=D=

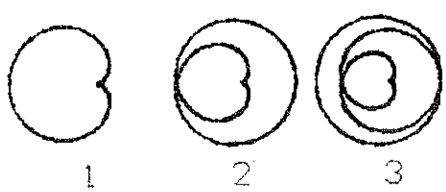


Fig. 12 Epicycloids: A=1, B=D=

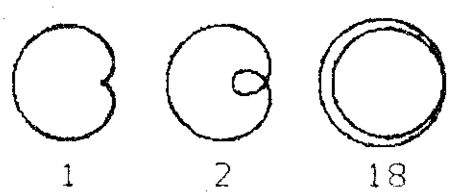


Fig. 13 Epicycloids: A=B=1, D=

It is useful to be able to transform the system of parametric equations of the cardioid into corresponding equations in polar coordinates. If A = B and D = nA, Eqs. (12) become

$$X = 2A\cos\Theta - nA\cos(2\Theta)$$
$$Y = 2A\sin\Theta - nA\sin(2\Theta)$$

Using the trigonometric double angle formulas yields

$$X - nA = 2A(1 - n\cos\Theta)\cos\Theta$$
$$Y = 2A(1 - n\cos\Theta)\sin\Theta$$

Let $r^2 = (x - nA)^2 + y^2$

Thus

$$r = 2A(1 - n\cos\Theta) \qquad (13)$$

This result may be illustrated by Fig. 14 in which a difference nA between two coordinate systems can be seen. If n = 1 in Eq. (13), the graph is shown in the first figures of Figs. 12 and 13.

Transformation of Cyclic Curves

Case I

Replacing B with -B in the equations of the hypocycloid, the

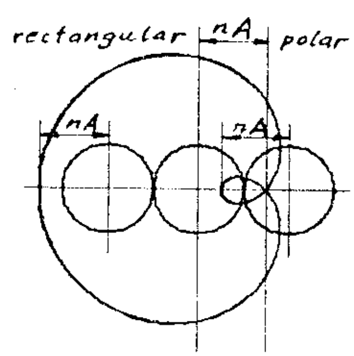


Fig. 14

epicycloid is formed but must be rotated through angle E.

In Fig. 15, an angular displacement between two similar patterns occurs. To determine this displacement angle E, replace B by -B in Eqs. (1) to obtain

$$X = (A + B)\cos\Theta + D\cos\alpha$$
$$Y = (A + B)\sin\Theta + D\sin\alpha \quad (14)$$

where $\alpha = (A + B)\Theta/B$

The coefficients in Eq. (12) are negative and in Eq. (14) they are positive. It is possible to show that a rotation of the Eqs. (14) through an angle E causes the signs of D to change from positive to negative. Substitution of Eqs. (14) into

$$X1 = X\cos E + Y\sin E$$
$$Y1 = -X\sin E + Y\cos E$$

yields

$$X1 = (A + B)\cos(\Theta - E) + D\cos(\alpha - E)$$
$$Y1 = (A + B)\sin(\Theta - E) + D\sin(\alpha - E)$$

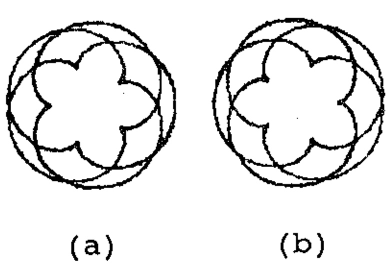


Fig. 15 (a) Hypocycloid: A=5, B=-3, D=3
(b) Epicycloid: A=5, B=3, D=3

For

$$\Theta1 = \Theta - E$$

and

$$\alpha1 = (A + B)\Theta1/B$$

then

$$\alpha - E = \alpha1 + AE/B.$$

Letting $AE/B = \pi$ and using the trigonometric simplifications

$$\cos(\alpha1 + \pi) = -\cos(\alpha1)$$

and

$$\sin(\alpha1 + \pi) = -\sin(\alpha1)$$

yields

$$X1 = (A + B)\cos(\Theta1) - D\cos(\alpha1)$$
$$Y1 = (A + B)\sin(\Theta1) - D\sin(\alpha1)$$

where $\alpha1 = (A + B)\Theta1/B$

This is a standard form of an epicycloid of which the condition is $AE/B = \pi$ or

$$E = B\pi/A \quad (15)$$

Consider also the transformation from epicycloids into hypocycloids. The same result may be obtained as in Eq. (15) for Eqs. (12) in which replacing B by -B in a way that parallels the work done for Eqs. (1). This is similar to Fig. 15 transforming from left to right by rotating the axes through the angle $B\pi/A$, as illustrated in Fig. 16.

Case II

If B and D are replaced by -B

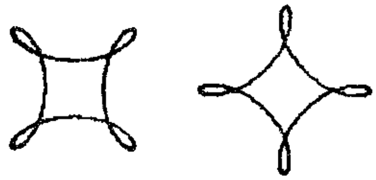


Fig. 16 (a) Epicycloid: A=4, B=-3, D=2
(b) Hypocycloid: A=4, B=3, D=2

and -D, respectively, in Eqs. (1), Eqs. (12) are formed directly and vice-versa. Hence, Eqs. (1) and (12) may be combined as

$$X = (A + B)\cos\Theta - D(B/|B|)\cos[(A + B)\Theta/B]$$
$$Y = (A + B)\sin\Theta - D(B/|B|)\sin[(A + B)\Theta/B]$$

These equations represent a hypocycloid if B is negative, an epicycloid if B is positive, and

if D = |B|, the shape has generalized curves
if D < |B|, the shape has shortening curves
if D > |B|, the shape has extended curves

Fig. 17 illustrates some patterns which are formed by combined equations.

Conclusions

The hypocycloid, epicycloid, and rose curves are included among spirographs, for all of them can be transformed from an equation given in one form into the corresponding equation in another form.

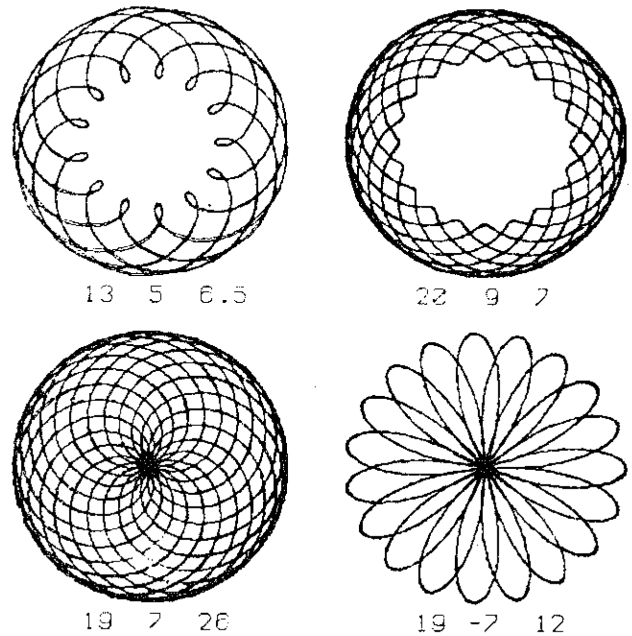


Fig. 17 Combinations, A=, B=, C=

The fundamental equations of a spirograph are Eqs. (1). According to different values of A, B, and D, different figures may be created.

If A and B are mutually prime in a hypocycloid, A is the total number of blades. If B is replaced by A - B, the hypocycloid may have similar figures.

If D = A - B in Eqs. (1), the graph is a hypocycloid passing through the center of the pattern. It is convenient to be able to transform the parametric equations with $n = A/(2B - A)$ into the corresponding equations of rose curves and vice versa.

If B is replaced by -B in the equations of the hypocycloid, the epicycloid is formed, but it must be rotated through an angle $B\pi/A$ and vice versa.

Computer Graphics and the Development of Visual Perception in Engineering Graphics Curricula

Scott E. Wiley

Department of Technical Graphics
Purdue University
West Lafayette, Indiana

Like all educational goals, the development of visual perception must be purposely designed into curricula. Visual perception is defined, the need to develop it is described, and methods which engineering graphics educators can use to develop it are outlined in the context of new computer graphics technology integrated throughout a revised curriculum.

The Continuing Revision of Engineering Graphics Curricula

Engineering graphics is in a transition phase in which many educational practices are being challenged by new technologies and increased scholarly activities. Many have recognized that the curriculum of engineering graphics is due for adjustment. As only one indicator, NSF, through SIGGRAPH, has funded a study to revise engineering graphics curricula for the 1990's and beyond[1]. It is hoped that this article will contribute to changes which might result from the SIGGRAPH grant as well as inform members of EDGD of topics previously covered by the author and new information related to it.

The focus of this article is how on computer graphics can be integrated throughout the engineering graphics curriculum to increase visual perceptual skills. A review of foundational material will lead toward conclusions which demonstrate how viewer-controlled animation and other methods can be used to develop visual perception ability.

A Review of Definitions of Visual Perception

As indicated by Wiley[2], engineering graphics educators frequently encounter students who have difficulty visualizing two and three dimensional drawings both during and after the solution of graphics problems. This common visualization problem has been identified by educators and psychologists as a visual perception deficiency which can be corrected through a sequential and detailed approach to the learning process. To review the problem and to comprehend how computer graphics can help, a review of definitions of visual perceptual development is necessary. Visual perception is the ability to:

A. Comprehend our environment

through the neurovisual system[3].

B. "See" or understand what one is observing[2].

C. Visualize possible changes in what one is observing[2].

Competent visual perception ability might enable an engineering graphics student to determine a missing view mentally after comprehending two given views, without using a drawing sequence as a mandatory visualization crutch[3]. Mentally determining a missing view by visualization would be regarded by Bloom[4] as a higher-level skill than drawing the correct missing view because cognitive processes necessarily precede psychomotor processes. Teaching higher level cognitive skills, such as visual perception, should then take precedence in our curricula. The key problem, though, has been identifying effective teaching practices which develop visual perception.

Visual Perception, Teaching Methods, and the Effectiveness of Computer Graphics

Many authors hold that the development of visual perception is a key area in need of revision. Wiley[2] considers it a "missing goal" and has suggested various ways of amending curricula to meet that goal by using traditional freehand and mechanical equipment. It has also been noted that teaching practices must actively teach it throughout an entire course or curriculum, instead of passively relying on a "Goss Box" or mechanical drawing sequences. Figure 1 provides a hierarchical visual perception learning sequence which could be followed in a traditional mechanical drawing course.

Computer graphics has been identified as a potent alternative method, but it should be pointed out that computer graphics is more than electronic drafting; it is a powerful visualization tool. Familiarization with introductory computer processes and equipment will not necessarily develop visual perception any more than familiarization with mechanical processes.

Computer graphics, especially high-end computer graphics, offers levels of realism and flexibility far beyond those provided through mechanical drafting. Objects can be viewed in a variety of ways simultaneously, edited quickly, and animated. They can be viewed orthographically and pictorially at the same time, thereby, providing increased opportunities for visual comparison. This is of great value.

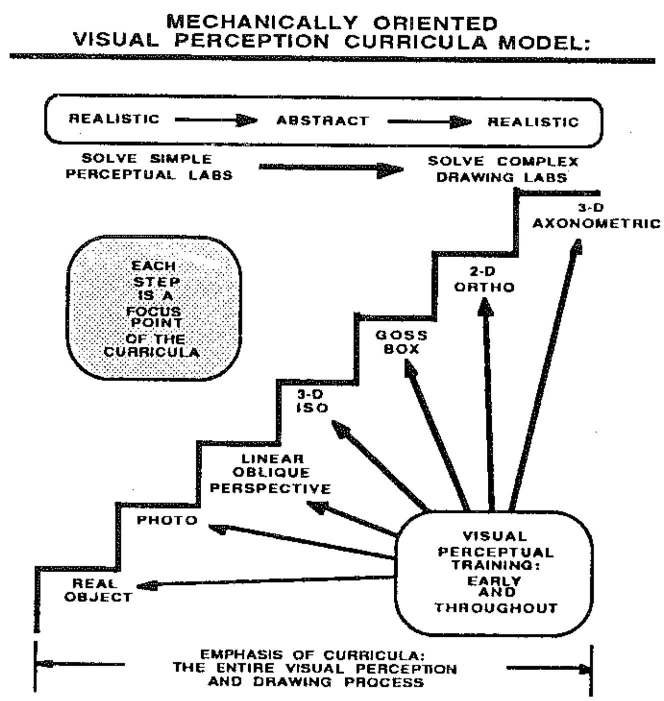


Figure 1

Real objects are normally perceived three dimensionally and are compared against themselves when picked up, held, and rotated. Computer graphics provides this same opportunity to increase visualization by allowing a 3D model to be visualized in several ways as it is being constructed. Multiple colors, textures, values, and geometric surface features can be presented to the viewer and edited quickly in the process. As with handheld objects, edges shear, colors and values change, and out-of-view surfaces can rotate into view. Through these dynamic interactions, many characteristics are presented simultaneously to build a complex perception of the object. By contrast, mechanical drawings are slowly produced, harder to edit, and present limited features, usually only linear features. Full color 3D computer animators can simulate most of the characteristics of real objects. Students can rotate objects and exert dynamic control over the perceptual learning process in real time.

A Computer Graphics Curricula Model for Developing Visual Perception

The mechanically oriented perceptual model (Fig. 1) provides non-automated steps which can build visual perception, but a computer oriented curriculum can provide the added potential of animation and viewer control of realistic objects. Figure 2 shows an example of a computer-oriented visual perception learning model. Throughout each computer-generated step the student can have full control of the perceptual and electronic drawing process. At any point the process can be slowed or reversed to reinforce perceptual learning experiences. Note should be taken that this model places emphasis on realistic representations of 3D objects early within the learning sequence. This is a contrast to many electronic courses which tend to lead students from 2D to 3D with 3D experience usually limited to harder to perceive wireframe models. In the model suggested, wireframe models follow, rather than precede, solid models. And as indicated, the curriculum highlights visual perception throughout the entire electronic drawing sequence. In this manner the ambiguities of wireframes are resolved by providing prior experience with related and easier to perceive engineering objects.

Viewer Controlled Animation, Motion Parallax, and Visual Perception Development

Some studies indicate that the development of specific visual skills is better achieved through very specific teaching practices[5,6,7,8]. As noted before[2], motion parallax is the ability to perceive apparent changes in the location of an object as a result of continuing changes in its position in space or continuing changes in the position from which it is viewed. Dorethy revealed that specific analysis of animations containing multiple depth cues significantly improved depth perception and was found to be more effective in developing depth perception than static visuals which also contained multiple depth cues. Apparently, the development of visual skills does not have to be drawing oriented to be effective. Visual analysis of animations alone can be effective.

Principles from Dorethy's study apply to the development of depth perception by engineering graphics students. Similar to the Roadrunner cartoons used in Dorethy's study, the motion parallax displayed during computerized solid modeling animation could provide increased visual insight into static 2D and 3D relationships, auxiliary views, perspective, and axonometric illustration. If the possibility of viewer control is added, students could rotate geometric structures at will so as to be able to perceive real time changes as necessary. The virtual space the object occupies could also be comprehended through checking displayed XYZ coordinates of any selected feature of the rotating object. As the student rotates the displayed object, coordinates would reflect a selected point's relative position with respect to a reference point. Like the Roadrunner cartoon, edges would shear and features could enlarge and reduce, appear and disappear, but all of these would take place inside the viewer-controlled, computer-indicated virtual space. And, unlike the cartoon, the geometric object could be more complex either in amount or kind of surface information.

But what can one do if the hardware required for viewer controlled solid modeling animation cannot be obtained? A solution might be to secure a low cost workstation and project animations onto an overhead screen with an LCD displayer. What is forfeited is viewer control, but what is gained is the ability to display an object to an entire class that is viewed in nearly the same way by everyone. The projection effect of an apparent 3D object onto an enlarged 2D screen causes object-to-background shearing to be the same no matter from where it is viewed. In real terms this means that group perception of an object can be controlled so that discreet elements are highlighted individually for enhanced perception.

Conclusions

Advancing computer graphics technology promises to accelerate the need for visual knowledge. At the same time it provides answers to long standing problems. If educators are willing to

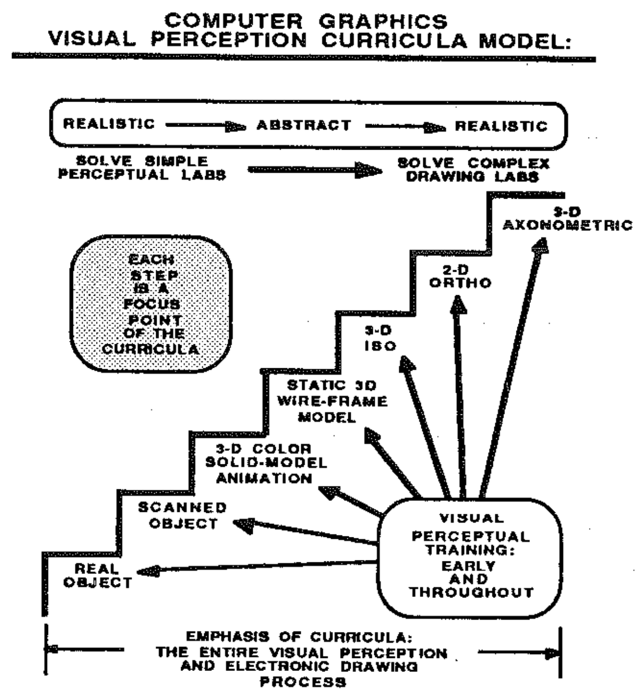


Figure 2

adapt, their curricula can be revised to take advantage of the visual perceptual learning power made possible by advances in areas such as 3D color solid modeling and animation. The question, however, is whether or not advantage will be taken of advancing technology and whether older teaching methods will be revised.

Accepting the development of visual perception as a dominant curricula goal may be the beginning of that necessary change. Advances in computer graphics have already made the computer oriented perceptual model (Fig. 2) a possible reality. But, possibilities do not become realities until applied.

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Chairman's Message

by
Frank Croft

It doesn't seem possible but this is the third and last Chairman's Message that I have the privilege to write. The past year has been filled with many challenges and I discovered some very positive things about myself and the Division in facing these challenges. I am convinced that the EDGD members compose the most outstanding group of professionals within ASEE. I wish to thank you all for the leadership opportunity that I have had over the past year. It has been a very enjoyable year and I look forward to continuing the work of the Division with all of you in the coming years.

The Annual ASEE Conference in Toronto is only a few weeks away and I sincerely hope all of you are making plans to attend. This conference and the EDGD program, I believe, will be outstanding. Fritz Meyers has worked very hard to organize the program and the advanced word is that the presentations will be excellent. We all owe Fritz a well-deserved "Thank you" for his efforts. If you missed the Mid-year Conference in Tuscaloosa, Alabama, you really did miss something - don't miss the ASEE Annual Conference in Toronto or you will miss something very special.

Toronto is a cosmopolitan city with a unique character that is difficult to describe - IT MUST BE EXPERIENCED! I have had the pleasure of visiting this extraordinary city several times. It is one of the cleanest metropolitan areas in North America and the Canadian people are indeed special. The Royal Ontario Museum is outstanding and houses a very large collection of dinosaur fossils. It should be on your list of side trips and is not far from any of the conference hotels. Also, the CN Tower is another attraction that is worth seeing. From the observation deck on a clear day, one can see the U.S. across Lake Ontario as well as many other sights. With luck, the Blue Jays may be playing baseball in the new stadium on the lakefront. Toronto has much to offer and everyone should enjoy their stay in this fascinating city. I certainly am looking forward to this conference and I hope to see you there.

1990 ASEE Annual Conference

by
Fritz Meyers

"Bienvenue a Canada". Welcome to Canada and to Toronto, the most cosmopolitan city in North America. Toronto is a big city - an exciting city - a clean city with outstanding public transportation. It also has big city prices - about the same as New York City, but US citizens do get the 15% advantage in exchange rates. Parking at the hotels (Sheraton Centre and Towers, Holiday Inn Downtown, and the Hotel Intercontinental), is approximately $18/day; hotel breakfasts are about $12. However, under the hotel is a vast network of underground shops, stores, and eating places stretching for nearly two miles! Variety is infinite and prices are less shocking. Accommodations are also available at the University of Toronto and Victoria College.

Our meals at official meetings will be standardized within each hotel, i.e., all Tuesday dinners at the Sheraton Centre will have the same menu. This will allow ASEE to have a single blanket guarantee for numbers of people and will help reduce the cost of official meals. You will want to eat outside the hotels for other meals - Toronto has more than 7000 restaurants.

Canada is a foreign country, however, a very friendly one. Customs should not be too much of a problem. We were given two caveats at the planning meeting. Don't bring electronic equipment into Canada - you can get it in, but it is very difficult to get it back into the US. Secondly, faculty members who are not US citizens must have a green card or their passport with a visa for this trip to Canada. No visa or no green card and a foreign national will not be allowed to reenter the USA.

The schedule for the week is a different format than last year because the Canadian Conference on Engineering Education is meeting concurrently with ASEE; a joint major plenary session followed by three breakout sessions will occupy all of Wednesday morning. The mini-plenaries will be held from 10:30 to noon on Tuesday; no other sessions may be held at that time. Likewise, the period from 2:30 to 4:15 on Monday is reserved for visiting the exhibits.

Sessions sponsored by the EDGD include:

1237 - Mon., June 24, 8:30-4:15
Creative Engineering Design Competition
William C. Koffke, Villanova
A display of engineering design projects by students from the US and Canada. Students at all levels may enter. Awards for entries, judged primarily on creativity and presentation, will be made at the EDGD Banquet.

1238 - Mon., June 24, 8:30-10:15
Methods to Improve Visualizations Skills I
Ileana Costea, Cal. State - Northridge
Visualization skills are important to all engineers. Research in methods to improve visualization will be presented: both traditional and new techniques. Some statistical studies of performance on visualization tests

will also be presented. This is the first of two related sessions.

1338 - Mon., June 24, 10:30-12:00
Methods to Improve Visualization Skills II
Edward Knoblock, Univ. of Wisconsin - Milwaukee
(See session 1238.)

1638 - Mon., June 24, 4:30-6:00
EDGD Executive Committee Meeting
Frank M. Croft, The Ohio State Univ.
Business meeting of the officers and directors. Open to all interested members of the division.

2237 - Tues., June 25, 8:30-4:15
(See session 1237.)

2238 - Tues., June 25, 8:30-10:15
Standards for Engineering Design Graphics Communication
Larry Goss, Univ. of Southern Indiana
Speakers will present information and research on standards for visual and electronic communication of design information. Both existing and proposed standards will be discussed.

2438 - Tues., June 25, 12:30-2:00
Curriculum Standards for Engineering Graphics I
Rollie D. Jenison, Iowa State Univ.
Two major research programs and several individual studies are questioning current curricula in engineering graphics. The questions and the proposed solutions offered by major programs and individual research will be presented and discussed. This is the first of two related sessions.

2538 - Tues., June 25, 2:30- 4:15
Curriculum Standards for Engineering Graphics II
Edward D. Galbraith, California State Polytechnic University
(See session 2438.)

2738* - Tues, June 25, 6:30-10:00
Annual Awards Banquet - $28
Frank M. Croft, The Ohio State University
The program will include presentation of the Division Distinguished Service Award and awards for the Creative Engineering Design Competition. Social hour with hosted bar will precede the meal.

3437 - Wed., June 26, 12:30-4:30
(See session 1237.)

3438* - Wed., June 26, 12:30-2:00
Annual Business Luncheon - $13
Frank M. Croft, The Ohio State University
Open business meeting for all members of the division.

Session locations were not available at the time of publication. ASEE will publish this information in the conference program.

* Denotes meal event

Fourth International Conference on Engineering/Computer Graphics and Descriptive Geometry

Sponsored by ASEE's Engineering Design Graphics Division and the Florida International University. Miami, Florida.

For technical information contact:

Dr. Oktay Ural
ASEE Computer Graphics Conf.
Dept. of Civil and Environmental Engrg.
Florida International Univ.
Miami, FL 33199

Ph. (305) 348-2824
FAX (305) 348-2802

General Information:

This conference will be a continuation of the international conferences held in Vancouver - 1978, Beijing - 1984, and Vienna - 1988.

Topics include:

Theoretical graphics
Engineering computer graphics
Graphics teaching
Other applications of geometry

More than 100 papers by authors from 20 countries will be presented. The conference will be an opportunity to meet outstanding experts from all over the world.

Conference Location:

The Hyatt Regency Hotel Miami is a riverfront showcase. All rooms have balconies so that guests may enjoy the sun and cool breezes blowing off the waters of Biscayne Bay. Its location in the heart of downtown Miami affords one the enjoyment of fine restaurants, cultural activities, shopping, and many attractions.

Address:
Hyatt Regency Miami
400 SE Second Avenue
Miami, FL 33131-2197
ATTN: Reservations

Ph. (305) 358-1234
TELEX# 415316
FAX# (305) 358-1234 EXT 3138

Preliminary Conference Schedule:

Monday, June 11, 1990

5:00-7:00 pm	Registration
7:00-8:00 pm	Reception

Tuesday, June 12, 1990

8:30-10:00 am	Opening Ceremony
10:30 am-Noon	Plenary Session
Noon-2:00	Conf. Luncheon
2:00-5:00 pm	Technical Sessions and Exhibits

Wednesday, June 13, 1990

9:00 am-Noon	Technical Sessions
2:00-5:30 pm	Technical Sessions and Exhibits

Thursday, June 14, 1990

9:00 am-Noon	Technical Sessions
2:00-5:30 pm	Technical Sessions and Exhibits

Friday, June 15, 1990

9:00 am-Noon	Bus Tour of Miami (reservation required)

Calendar of Events

by
Bill Ross

1990 4th International Conference on Engineering and Descriptive Geometry
June 11-15, 1990
Miami, Florida
(See previous article)

1990 Annual ASEE Conference
June 24-28, 1990
Toronto, Ontario, Canada
Host: University of Toronto
EDGD Prog. Chr.: Fritz Meyers
The Ohio State University
(614) 292-1676

1990-91 EDGD Mid-year Conference
November 18-20, 1990
Tempe, Arizona
Host: Arizona State Univ.
General Chair: Del Bowers
Arizona State Univ.
(602) 965-3165
Prog. Chair: Gary Bertoline
The Ohio State University
(614) 292-6230

1991 ASEE Annual Conference
New Orleans, LA

1991-92 EDGD Mid-year Conference
Host: Old Dominion Univ.
Norfolk, VA

1992 ASEE Annual Conference
Toledo, OH

1992 5th International Conference on Engineering and Descriptive Geometry
August 17-21, 1992
Melbourne, Australia

1992-93 EDGD Mid-year Conference
San Francisco, CA (tentative)

International Computer Graphics Calendar

by
Vera Anand

Jun 6 - 8, 1990
Sixth Annual Symposium on Computational Geometry, Berkeley, CA, Contact: Frances Yao, Xerox Palo Alto Research Center, 3333 Coyote Hill Rd., Palo Alto, CA 94304.

Jun 11 - 15, 1990
Fourth International Conf. on Computer Graphics and Descriptive Geometry (See page 49).

Jun 26 - 30. 1990
Computer Graphics International 1990, Singapore. Contact: Victorine Toh, CGI '90 secretariat, Institute of Systems Science, National University of Singapore, Kent Ridge, Singapore 0511. Ph. (65) 772-2003.

Jul 8 - 12, 1990
CATS '90 - Internat. Conf. on Computer Aided Training in Science and Tech., Barcelona, Spain. Contact: Prof. E. Onate, Centro Internacional de Metodos Numericos en Ingenieria, Jorge Girona Salgodo, 31. 08034 Barcelona, Spain Ph. 34-3-205 70 16/204 82 52.

Aug 6 - 10, 1990
SIGGRAPH 90, Dallas, TX, Contact: David D. Loendorf. Ph. (505) 665-0866.

Aug 27 - 31, 1990
INTERACT '90, Cambridge, England - an international conf. in the area of human-computer interaction. Contact: Karyn McCart-

ney, INTERACT '90, The British Computer Society, 13 Mansfield St., London, W1M 0BP, England. Ph. 44 (0)1 637 0471.

Aug 28 - 30, 1990
ICED 90, International Conf. on Engineering Design, Dubrovnik, Yugoslavia. Contact: HEURISTA, Conf. Dept., Postfach 102, CH-8028 Zurich, Switzerland.

Sep 3 - 7, 1990
Eurographics '90 - a conf. and exhibition sponsored by the European Assoc. of Computer Graphics, Montreaux, Switzerland. Contact: Eurographics '90, Conf. Secretariat, Paleo Arts et Spectacles, Case postale 177, CH-1260 Nyon, Switzerland. Ph. (41) 22 62 13 33.

Sep 10 - 14, 1990
AUSGRAPH 90, the eighth annual conf. of the Australasian Computer Graphics Assoc., Melbourne, Australia. Contact: AUSGRAPH 90 secretatiat, P. O. Box 29, Parkville, VIC 3052, Australia. Ph. (03) 819-8124.

Dec 9 - 11, 1990
Workshop on Volume Visualization, San Diego, Ca. Contact: Nick England, Sun Microsystems, Inc., P. O. Box 13447, Research Triangle Park, NC 27709-3447. Ph. (919) 469-8300.

For further information, contact Vera Anand, 302 Lowry Hall, Clemson Univ., Clemson, SC 29631. (803) 656-5755

Call for Papers

EDGD 1990-91 Mid-year Conference
November 18-20, 1990
Arizona State University
Tempe, AZ

Visualizing Engineering Design Graphics in the 90s: The Gateway to the 21st Century

Suggested topics:

Engineering design graphics in the 21st century

Curriculum issues for the 90s

Sketching in the EDG Curriculum: A renewed emphasis

Engineering Design Graphics: An historical perspective

A research agenda for EDG

Visualization applied to EDG

Future trends in hardware and software

Abstracts of 250 words are due July 1, 1990. Submit to:

Gary R. Bertoline
Program Chair
Dept. of Engrg. Graphics
The Ohio State Univ.
240 Hitchcock Hall
Columbus, OH 43210

(614) 292-7930

Notice of acceptance mailed by July 30, 1990. Completed papers for Proceedings are due Sep 15, 1990.

Call for Papers

on

Geometric Modeling in Engineering Education

for the Winter, 1991 issue of the *EDG Journal*

sponsored by the EDGD Geometric Modeling Committee

The purpose of this issue is to provide a number of articles dealing with educational as well as research and application issues in the emerging field of geometric modeling in engineering. Topics of interest include:

Curricula in geometric modeling at the undergraduate and graduate levels in engineering

Experience and techniques of teaching geometric modeling courses

Innovative ideas for teaching geometric modeling in the freshman graphics course

Geometric modeling applications in the various areas of engineering

Other topics on geometric modeling and design

Submit papers to:

Prof. Nadim Aziz
Engrg. Graphics Program
320 Lowry Hall
Clemson Univ.
Clemson, SC 29634

not later than July 1, 1990.

Call for Papers

1991 ASEE Annual Conference
New Orleans, Louisiana
June, 1991

Topics: Applied methods and research in visualization, engineering graphics curriculum development, applied computer graphic and CADD teaching techniques, emerging standards in engineering graphics and CADD, computer based geometric modeling, and engineering graphic communication and documentation.

Deadlines:

Abstracts due: Oct. 15, 1990.

Full papers due: Dec. 10, 1990.
For inclusion in the ASEE Proceedings, papers must first be reviewed and accepted.

Contacts:

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(601) 325-3922

A Comment

by
Pat Kelso

I would like to second Bill Rogers' letter (Reflections from the 1989-90 Mid-year Meeting, Winter, 1990, Vol 54., No. 1) and extend the thrust, if I may, to include the paucity of purely graphics papers at mid-year conferences. Sometimes I think I am in *The Engineering Division of Canvasses, Polls, and Surveys*. At the risk of being overly presumptuous, I believe some within the division have ventured onto an unfruitful path because, basicly, they do not accept that the quicker the virtual space of the computer replaces the virtual space of multiview projection, for purposes of simulation, the better for all concerned. I suggest the only use for a 2-D surface now is for sketching and communication. (Want something exciting? Tell your department head that you have developed a descriptive geometry course for computer geometric modeling!) And even communication is not what it was. With automatic plot-outs of viewports, communication no longer requires *drawing*. Even for details, such as the correct treatment of crossing (do they intersect of not?) hidden lines. If there is an ambiguity, the computer quickly produces a view resolving it - with shading from whatever light direction one wants. The virtual space of the computer also makes superfluous the teaching of, dare I say it: *VISUALIZATION*. In computer geometric modeling, students instinctively learn visualization - en passant, as it were. Singleview wireframes are visualization mind benders (actually, they are impossible to visualize in a single view, so students must keep spatial orientation in mind from step to step: ergo, student visualization enhancement). And if need be, the computer produces a shaded pictorial at any stage of the students' modeling process. Not that I think visualization can be taught. (Improved, yes, but, alas, it is, in the final analysis, an aptitude.) Rhapsodizing about visualization may be dangerously close to a cop-out for more meaty matters. Perhaps it is best included only with religion, politics, and economics (especially, economics).

It seems to me we are in a unique position, time-wise, as engineering graphics instructors. It will be several years before current freshmen hit the heavy design courses; our obligation, I suggest, is to give "Kentucky windage" to these students. I personally believe instructors of engineering graphics are in a better position to make this decision than our learned upper level colleagues. Most of us are up to speed on the latest graphics innovations and many upper level teachers have not made the complete transition to computer geometric modeling for their courses - but they will have by the time current freshmen reach them.

Editor's Comments

by
Barry Crittenden

A procedure for the review of papers to be published in the Proceedings of the ASEE Annual Conference was used for the first time this year. Of the twenty-four persons presenting papers in Toronto this June, only fifteen submitted papers for review. These fifteen authors had the option of having their papers reviewed only for the ASEE Proceedings, or for both the ASEE Proceedings and the *EDG Journal*.

From the standpoint of the *EDG Journal* editors and the EDGJ Board of Review, the process went relatively smoothly. The schedule followed for this review process was as follows:

Nov. 15 - Abstracts due (to program chairman)

Dec. 1 - *EDG Journal* Technical Editor notified of prospective participients

Dec. 10 - Technical editor informs prospective participants of review procedure

Jan. 10 - Papers due (to Technical Editor)

Jan. 15 - Papers sent to reviewers

Jan. 15 - Session chairs notified of session participants

Mar. 1 - Authors notified of review results. ASEE notified to send mats to authors whose papers were accepted for the Proceedings

Apr. 1 - Completed papers due at ASEE headquarters

There is always room for improvement. Please send your suggestions for a better review procedure to:

George R. Lux
Technical Editor, EDG Journal
EF, VPI&SU
Blacksburg, VA 24061-0218

(703) 231-6555

as soon as possible. Suggested changes will be discussed at the EDGD Executive Committee Meeting in Toronto on June 24th.

Reviewers for the *EDG Journal* are wanted. If you are willing to devote several hours per month reviewing papers, please notify George Lux (address above).

Temporary Position Wanted

In descriptive geometry, photogrammetry, and engineering graphics. Thirty-four years of teaching and research experience. Contact:

Prof. Wagih N. Hanna
Prof. of Descriptive Geometry
Faculty of Engineering
Ains Shams Univ.
Abbasia, Cairo, Egypt

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This Journal is devoted to the advancement of engineering design graphics technology and education. The Journal publishes qualified papers of interest to educators and practitioners of engineering graphics, computer graphics, and subjects related to engineering design graphics in an effort to (1) encourage research, development, and refinement of theory and application of engineering design graphics for understanding and practice, (2) encourage teachers of engineering design graphics to experiment with and test appropriate teaching techniques and topics to further improve the quality and modernization of instruction and courses, and (3) stimulate the preparation of articles and papers on topics of interest to the membership. Acceptance of submitted papers will depend upon the results of a review process and upon the judgement of the editors as to the importance of the papers to the membership. Papers must be written in a style appropriate for archival purposes.

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Submit complete papers, including an abstract of no more than 200 words, as well as figures, tables, etc. in quadruplicate (four copies) with a covering letter to J. B. Crittenden, Editor, Engineering Design Graphics Journal, EF - VPI&SU, Blacksburg, VA 24061. All copy must be in English, type double-spaced on one side of each page. Use standard 8 1/2 x 11 inch paper only, with pages numbered consecutively. Clearly identify all figures, graphs, tables, etc. All figures, graphs, tables, etc. must be accompanied by a caption. Illustrations will not be redrawn. Therefore, ensure that all line work is black and sharply drawn and that all text is large enough to be legible if reduced to single or double column size. High quality photocopies of sharply drawn illustrations are acceptable. The editorial staff may edit manuscripts for publication after return from the Board of Review. Galley proofs may not be returned for author approval. Authors are therefore encouraged to seek editorial comments from their colleagues before submission of papers.

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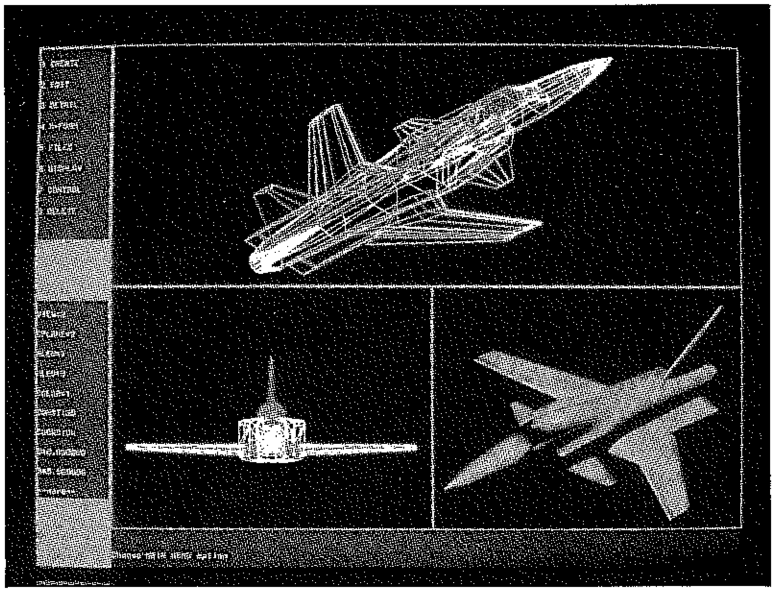

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