Representing the Past by Solid Modeling + Golden Ratio Analysis

Suining Ding
Indiana University Purdue University Fort Wayne

Abstract

This paper describes the procedures of reconstructing ancient architecture using solid modeling with geometric analysis, especially the Golden Ratio analysis. In the past the recovery and reconstruction of ruins required bringing together fragments of evidence and vast amount of measurements from archeological site. Although researchers and scholars in the archeological field have used traditional manual methods to obtain data, the innovative application of solid modeling and geometric analysis techniques can help verify and reveal unknown or uncertain data. This paper demonstrates application of solid modeling with an approach from geometric construction perspective, along with data collection, literature review and photography analysis. Two case studies are demonstrated in this paper. They are the reconstruction of ruined ancient luxury complex Baths of Caracalla and building of a 3D digital model of Florence Cathedral. In addition, several solid models of case studies are presented in the paper.

1. INTRODUCTION

The objective of this study is to explore solid modeling and geometric analysis, especially the Golden Ratio analysis, for reconstructing historical architecture using 3-D AutoCAD. Two case studies are demonstrated in this paper. They are the reconstruction of ruined ancient luxury complex Baths of Caracalla and building of a 3-D digital model of Florence Cathedral. The five steps involved in this project include data collection and literature review; geometric analysis, particularly the golden ratio analysis on floor plans, elevations and building structures; digital reconstruction of solid 3-D models; digitizing authentic architectural materials and decorative elements, and importing authentic materials into the materials library. The last step is to apply materials on the surface of the model and create lightings and perspective views. Finally, a summary of digital reconstruction procedure with geometric analysis, especially the golden ratio analysis, is presented in this paper.

Using CAD to build 3-D models is not a new task. Many designers and researchers have attempted this approach to build digital models to reconstruct buildings or cities that had ceased to exist. Digital media enables researchers and designers to derive better processes and methods in solving problems of historical architectural restoration and ancient ruins reconstruction. Tracing the past and discovering hidden treasures will provide designers and researchers with inspirations to create a sustainable future. Although there are many ways to reconstruct ruins and represent the past, including physical models, manual field data collections, or digital applications, more research still needs to be done in exploring new approaches. In this study, a hybrid application that combines digital methods, especially constructive solid modeling, with geometric analysis, along with data collection, literature review and photograph analysis, was explored.

The author has conducted preliminary studies in the area of reconstruction of ancient architecture with AutoCAD (Ding, March, 2005, June, 2005); this study is a further investigation with more defined objectives. In this study, AutoCAD software is used. Currently, there is no generic procedure available to guide designers and researchers to create 3-D architectural and interior space models with AutoCAD solid modeling techniques. One of the objectives of this research project is aimed specifically at the task of developing a set of procedures using AutoCAD solid modeling techniques and its Boolean operations for guiding designers and researchers to reconstruct ancient architecture. The significant contribution of this study is to provide the guidelines for solid modeling with geometric analysis to reconstruct historical buildings for architecture digital application and education.

2. LITERATURE REVIEW

2.1. RECONSTRUCTION OF ANCIENT ARCHITECTURE WITH DIGITAL METHODS

There exist several studies on digital modeling to reconstruct ancient architecture. Liu (Liu, 2003) explores a digital reconstruction procedure for historical architecture and cities which no longer exist by using 3D scanning, Virtual Reality cave technology and motion capture technology. As a case study, Serrato-Combe (Serrato-Combe, 2003) presented a theoretical reconstruction of the Aztec Templo Mayor in Mexico. He describes how a variety of digital approaches were used to grasp and appreciate the most significant architectural contributions of the early inhabitants of the Americas. Datta (Datta, 2003) demonstrated the application of a hybrid computational approach to the problem of recovering the surface geometry of early temple superstructures. The approach combines field measurements of temples, close-range architectural photogrammetry, rule-based generation and parametric modeling. It is evident from the literature review that although different digital methods have been utilized by researchers to reconstruct historical buildings and cities, different approaches still need to be explored, such as 3D AutoCAD solid modeling combined with the theory of geometric construction, as well as the golden ratio analysis.

2.2. CONSTRUCTIVE SOLID GEOMETRY (CSG) AND 3D AUTOCAD

Constructive Solid Geometry or CSG in short, is another way of representing solid objects. It provides a unique opportunity to make form transformation and space interlocking. Constructive solid geometry systems allow the definition of complex three-dimensional objects using a combination of simpler objects (Blackwell, 2000). CSG is used to build solid models by combing primitive objects with Boolean operations. Solid primitive objects, such as spheres, cubes and cylinders can be manipulated with Boolean Operations to produce new objects based on union, subtraction and intersection of primitive objects in CAD. In addition to Boolean operations, 3-D AutoCAD also has the ability to create complex solid geometry form by sweeping two-dimensional shapes and projections. The commands are REVOLVE and EXTRUDE. These are two very powerful commands in 3-D AutoCAD. Most of architectural components can be created by these two commands and Boolean operations. Therefore, 3-D AutoCAD with CSG method becomes a powerful tool to generate complex forms and achieve the goals of form and space interchange and manipulation.

2.3. GEOMETRIC CONSTRUCTION AND PROPORTION SYSTEMS

To represent ruins and historical building, one of methods can be used as reference in the recreation of the past is using digital models and geometric construction approach. Typically, there are two systems of proportions in ancient Greek and Roman architecture. One is the system based on the golden ratio, such as Le Corbusier's modular system. The Modular makes use of one particular mathematical relationship in design. It has had so much influence on design throughout history as

to deserve some special discussion (Pile, 2003). The ratio of the golden section is expressed in number .618 (Pile, 2003). The golden ratio can be described as if one divides a line into two unequal segments A and B so that the ratio of the first short segment A to the long segment B equals the ratio of the long segment B to the total line length (the short segment plus the long segment, A + B). The formula used for calculation is:

$$A / B = B / (A+B)$$

The other one is the system based on the square, apparently used by the Romans. In addition to square and rectangle, polygon, triangle, pentagon, and hexagon are popular forms used in Roman and Greek architecture. It is obvious from the review of literature that the square and its diagonal, the so-called ad quadratum, played a central role in the Roman architecture. Furthermore, a geometric construction involving the square called the sacred cut also seems to have been used by the Romans. The particular rectangle, the so-called roman rectangle, is another significant characteristic to the Roman's system of proportions in geometric construction.

Throughout history most architects have only incorporated geometry in the design using straightforward regular polygons and the circle. More than two millennia basilicas, domed and vaulted structures, have been generated principally by the projection or rotation of the three primary figures - circle, rectangle and triangle (Johnson, 1994). The Golden Ratio is a simple geometric construction, which can be used to shape windows and floor plans, to locate paving patterns and to divide courtyards. It is un-doubtable and quite evident that geometric construction has been used in architecture throughout ages. There is documented evidence that architects were aware of the Golden Ratio and possibly even its mathematics (Ostwald, 2000). Therefore, the Golden Ratio was widely used in geometric construction.

Another characteristic of Roman and Greek architecture is symmetry. Architects and builders have always sought systems of proportions. Marcus Vitruvius Pollio circa 70-25 BC, a Roman architect and engineer, wrote: "Symmetry is a proper agreement between the members of the work itself, and relation between the different parts and the whole general scheme, in accordance with a certain part selected as standard." (Vitruvius, 1960). Here Vitruvius uses symmetrical relationships to mean the same proportions, rather than some kind of mirror symmetry. Such a system would use the repetition of a few key ratios, to insure harmony and unity. It would have additive properties, so the whole could equal the sum of its parts, in different combinations. This would give a pleasing design, and maintain flexibility. Finally, since builders are most comfortable with integers, it would be based on whole numbers (Vitruvius, 1960).

In conclusion, existing literature indicates that AutoCAD with Constructive Solid Modeling is a powerful tool to create digital models. Although several experiments have explored digital methods to reconstruct historical buildings, no generic procedure using 3-D AutoCAD has been developed yet. Many historical buildings especially Roman and Greek architectures were built with geometric construction method. It makes the data collection and measurement much easier. In Roman and Greek architecture, repetitive geometric forms, symmetrical floor plans and elevations, as well as square and rectangular are found in almost every instance. One of the objectives in this research is to explore digital methods with mathematical geometry analysis approach to construct the ruins and historical buildings. The anticipated outcome of this study is to provide a generic set of procedures which combines a 3-D AutoCAD digital method and geometric analysis.

3. PROCESS

The author conducted a preliminary study with first hand data collections and digital photos as well as field measurements, and created a preliminary conceptual model. It is a portion of

Baths of Caracalla, which was created with the application of authentic Roman architectural materials. Based on that preliminary study, this research further investigated Baths of Caracalla and Florence Cathedral in order to create a more complex model with a set of generic procedures for using AutoCAD constructive solid geometry method. Internet search and literature review were conducted to find more accurate floor plans and elevations with graphic scales. The geometric construction methods were analyzed by mathematical calculations for both projects in order to verify uncertain data, such as floor plan dimensions and building heights, as well as the unknown portions of the structure. These two 3-D models were built by individual structural components, such as columns, triangular pediments, domes and vaults. These basic architectural components were created individually by using AutoCAD with the CSG method. Photoshop software was used to modify digital photos of authentic architectural materials so that higher quality images could be obtained. Authentic materials were imported into the materials library. The next step was to put all pieces together and apply authentic materials and lightings. Finally, different perspective views were generated. As an objective of this study, a set of generic procedures, especially for using 3-D AutoCAD with CSG methods, was generated.

***Figure 1**: 3-D Model Perspective of Baths of Caracalla (Ding, 2008)*

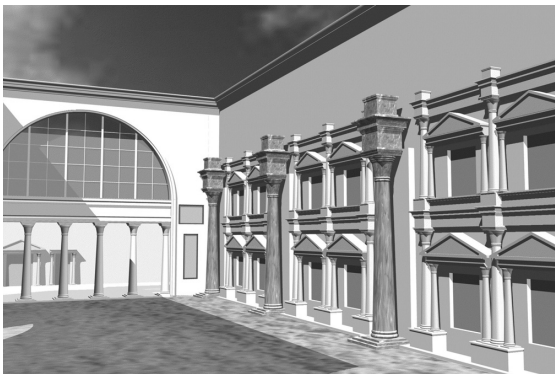

***Figure 2**: 3-D Model Perspective of Baths of Caracalla (Ding, 2008)*

2. RESULTS

Two digital models were created. One is the digital model of Baths of Caracalla (Figure 1, Figure 2 and Figure 3) and the other one is the digital model of Florence Cathedral (Figure 4, Figure 5 and Figure 6). Authentic Roman architectural materials have been applied to Baths of Caracalla. The following is a generic procedure for reconstruction of ancient ruins and historical buildings using digital methods with geometric analysis approach.

***Figure 3**: 3-D Model of Baths of Caracalla with Authentic Material (Ding, 2008)*

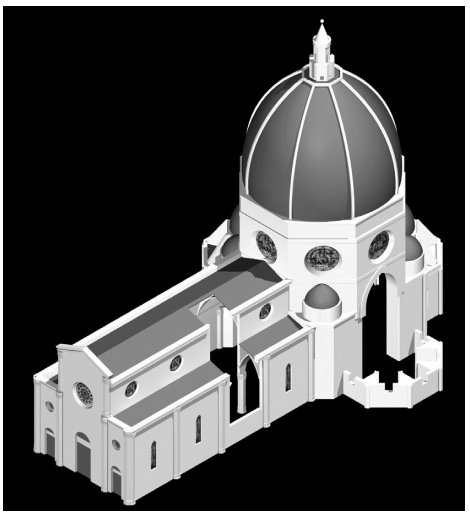

***Figure 4**: 3-D Model of Isometric View of Florence Cathedral (Ding, 2008)*

***Figure 5**: 3-D Model of Interior View of Florence Cathedral (Ding, 2008)*

***Figure 6**: 3-D Model of Interior View of Florence Cathedral with Barrel Vaults (Ding, 2008)*

Step One: Data collection and Literature Review. This is the first step for creating 3-D models. It includes field measurement and photo taking. However, it is not realistic for everyone to have the opportunity to physically go to the field for data collection. The literature review becomes crucial to find relevant information as much as possible. The Internet is a very useful source. Extensive amount of existing literature and photos can be found from the Internet. This confirms that mathematical geometry analysis is an indispensable instrument to verify uncertain data and to calculate unknown data. One recommendation for this step is that even though field measurements and photos were taken, it is still essential to do the literature review. The more information about the building or site that can be found, the smoother the process of creating the models will be. The second recommendation for literature review is to try to find floor plans and elevations with graphic scales. This allows one to enlarge or reduce the drawings as necessary without distorting the dimensions. Dimensions are critical for creating a more accurate 3-D model. Therefore, obtaining sufficient and accurate dimensions becomes a key for successful reconstruction process.

Step Two: Geometric analysis on floor plans and elevations. This step serves as verification for uncertain data such as dimensions and missing portions of the structure. It can also be used as a rationale for completion of floor plans and elevations when some of the structures are missing. A floor plan or elevation that is retrieved from the Internet or obtained from a book usually will not have all the dimensions that are shown. It is necessary to use the geometric method to calculate the unknown dimensions. As long as the drawings are presented with the graphic scale, it is not difficult to calculate all the dimensions. The following is an example of the geometric analysis for Florence Cathedral. Figure 7 is the floor plan and Figure 8 is the cross section of the cathedral. Literature review shows that squares and rectangulars were used extensively in Roman geometric construction. Circles on the floor plan and sec-

tion indicate that the shapes inside the circles are squares. It is obvious that many spaces were constructed following the rule of height being equal to the side length. If one side of the dimension is known, then the height will be found easily. The other characteristic of Roman architecture is the constant use of rectangle. By using the Golden Ratio, it is not hard to calculate the height of a rectangular in the section of the Cathedral (Figure 8). To apply the Golden Ratio in this case, in Figure 7, A is a longer segment and B is a shorter segment. The formula used for calculation of value for B is:

$$B / A = A / (A+B)$$

'A' can be found from the floor plan with a graphic scale. It is also the height for that square. Value of variable 'B' is the height of the rectangular, which is unknown. The value of variable 'C' can be found from the floor plan and C equals A + B. With a simple mathematic calculation, it is easy to find the value for B, which is about 3'-10". Another method is to use the number 0.618 and formula B / A = 0.618 to calculate the B value. The same process can be applied for the cathedral floor plan to calculate the value for variable D. The formula for variable D is D / A = A / (A + D). The third characteristic in Roman architecture geometric construction is symmetry. Adherence to this practice makes calculation and modeling process much easier and quicker. In Figure 6 of the section of the cathedral, portions on the right side are missing. But with the mathematical geometry analysis and the Golden Ratio theory, it is not difficult to figure out all the dimensions and reveal missing structure portions.

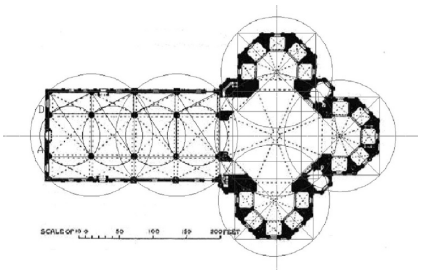

***Figure 7**: Florence Cathedral Floor Plan (Ding, 2008)*

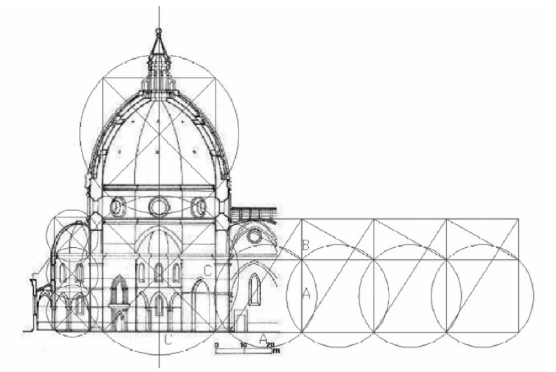

***Figure 8:** Florence Cathedral Cross-section (Ding, 2008)*

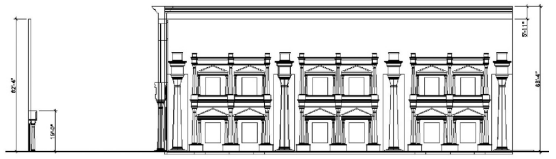

***Figure 9**: Elevation of Baths of Caracalla (Ding, 2008)*

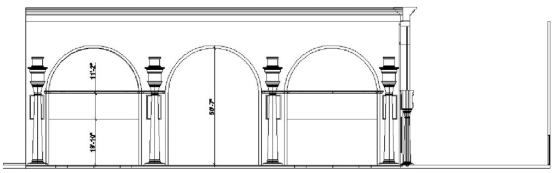

***Figure 10**: Elevation of Baths of Caracalla (Ding, 2008)*

Figure 9 and Figure 10 represent the elevations of the Baths of Caracalla. It is obvious that the building structure is symmetrical with repetitive geometric forms, symmetrical floor plans and elevations. These architectural characteristics make the model building process simpler. During the process, two major architectural components were created. They are column and triangular pediment. By using the symmetrical relationships theory of geometry, which means the same proportions, rather than some kind of mirror symmetry, it is easy to create a 3-D model of triangular pediments with different scales shown in Figure 12. You may just create one column and one triangular pediment and use SCALE com-

mand to create the 3-D models with a different scale. The digital model looks complex. But the process is very simple.

Step Three: Construction of digital 3-D models using CSG. After all the dimensions are obtained and verified with mathematical geometry analysis, it is the time to start building the digital model. The final perspective views of digital model look complex and hard to build. However, both Baths of Caracalla and Florence Cathedral are symmetrical and most architectural components are exactly the same or with a smaller scale. One may create a single model for a particular component and copy it over for the rest exact same components. The following describes the procedure and commands used to create the digital model. The digital method is 3-D AutoCAD with Constructive Solid Geometry (CSG). In other words, it is using 3-D AutoCAD with Boolean operations. The process is to create each individual architectural piece first and then put them together.

a. *Column (Figure 11) – Using REVOLVE command. First, draw half of the column elevation in 2-D based on the dimensions. Use PEDIT to join all single segments together to get a single entity before using REVOLVE command.*

Figure 11: *3-D Model of a Column (Ding, 2008)*

b. Triangular Pediment (Figure 12) – Use EXTRUDE command. Draw a triangle in 2-D, and then use PEDIT to join all segments together to make a single entity, and then extrude the polyline. Use SUTRACT Boolean operation to create recessed area on triangular pediment.

Figure 12*: 3-D model of Triangular Pediment (Ding, 2008)*

c. Dome (Figure 13) – Using REVOLVE command. The process is similar to creating the column. The differences are that you need to draw half of the section of the dome to present the thickness of the dome. The revolve axis can be offset from the edge of the section in order to create a void dome with open oculus, the unglazed opening on the top of the dome.

Figure 13*: 3-D Model of a Dome (Ding, 2008)*

d. Barrel vault (Figure 14) – Using EXTRUDE command. Draw the section of barrel vault in 2-D elevation, and then use PEDIT to

join all segments together to make it a single entity. Then extrude the single entity. For intersected barrel vault, copy the barrel vault just created and rotate it 90 degree and place it on top of the first one. Use UNION Boolean operation to create an intersected barrel vault.

Figure 14: *3-D Model of a Barrel Vault (Ding, 2008)*

e. These are the typical architectural components in classical architecture. If some special forms need to be created, CSG method (Boolean operations) is very powerful to achieve these goals.

Step Four: Digitizing authentic architectural materials and importing them into the materials library. After finishing the 3-D model, the next step is to apply authentic materials to the digital model. Photos from field or Internet, or scanned documents from books can be modified and edited by Photoshop or Photo Editor Software. When a photo image is not completed or needs some modifications, for example, the fresco of the Baths of Caracalla is not a completed image and some portions are missing, Photoshop can be used to modify the image to recover its complete look. Another example, the images of the stained glass in Florence Cathedral were retrieved from the Internet and modified by Photoshop. When the image is ready, you can import the images to the materials library, and then modify the materials by adjusting the bitmap.

Step Five: Applying authentic materials and lightings to the 3-D models and create perspective views. This is the last step and it is as straightforward as a standard process. One recommendation for this is to use distance lighting to simulate sunlight for exterior models because shadows will create dynamic effects on the images.

5. CONCLUSIONS AND RECOMMENDATIONS

The outcome of this study provides a systematic procedure of reconstructing ancient architecture with digital methods, such as Constructive Solid Geometry (CSG) and photo image editing. The process of this study demonstrates application of geometric analysis, especially the golden ratio analysis, to assist obtaining unknown data. The combination of digital methods and geometric analysis makes the entire process more precise and rational. It provides valuable recommendations to researchers who search for a method to reconstruct the past and need visualizations of ancient ruins and historical buildings. The limitation of this study is that this approach only applies to ancient buildings with geometric constructions. Future research could be a broader investigation, which includes other computational and mathematical methods.

Evidence of application of mathematics in architecture has been proven and studied in previous research works. The recommended future research could focus on the interdisciplinary study of mathematics in architecture aimed at mathematical analysis for reconstruction of the past with 3-D digital models. Built structures and their architectural representations are places where geometry, mathematics and construction discover their common nature. Therefore, a systematic investigation of using mathematical analysis as an instrument to aid obtaining unknown data and building dimensions is needed. It will provide valuable suggestions in creating digital 3-D models.

ACKNOWLEDGEMENTS

The author is grateful for the support of this work provided by Purdue Research Foundation's summer faculty research grant. All the models and drawings were created by the author with the support of this grant.

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