Using a Parametric Solid Modeler as an Instructional Tool

Kevin L. Devine
Illinois State University

Abstract

This paper presents the results of a quasi-experimental study that brought 3D constraint-based parametric solid modeling technology into the high school mathematics classroom. This study used two intact groups; a control group and an experimental group, to measure the extent to which using a parametric solid modeler during instruction affects student learning relating to the mathematical principles of areas and volumes of solids. The control group was taught using traditional instructional methods while the experimental group was taught using a combination of traditional methods and experimental methods utilizing a parametric solid modeler. Both the experimental and control groups were tested using written exams to measure the extent to which the experimental treatment affected student learning. While the experimental group performed better on a posttest, there was no statistically significant difference between the performance of the two groups.

INTRODUCTION

It is generally accepted that an understanding of the principles of geometry is very important in many engineering related disciplines. (Kent & Noss, 2000; Velichova, 2002; Wiebe, Branoff, & Hartman, 2003). Those who teach engineering graphics certainly recognize that the discipline relies heavily on an understating of the principles of geometry, and all too often instructors find it necessary to review the basics of geometry with their students. Several researchers have suggested that one primary instructional goal of engineering graphics should be the development of student understanding of Euclidean geometry (Bertoline, 1991; Wiebe et al., 2003). Towards that end, Wiebe, Branoff, & Hartman (2003) propose an instructional model for using parametric solid modeling software as a tool to help teach the principles of geometry in engineering graphics courses. Although needing further study, the primary focus of their instructional model addresses the principles of geometry as they relate to the dynamic behavior of solid models. Several sample exercises illustrate how a parametric solid modeler might be used to help students understand the relationship between dimensional and geometric constraints, and the resulting behavior of the model during subsequent editing. Their sample exercises utilize the ability of parametric solid modelers to provide feedback to learners that is both immediate and readily observable.

While the proposed instructional model of Wiebe, Branoff, & Hartman (2003) focused on dimensional and geometric constraints as applied in the discipline of engineering design, modern parametric solid modelers have an impressive array of tools that could be used to teach other geometry principles as well. For example, dimension equations might be used to illustrate the mathematical concept of proportionality. Boolean operations could be useful to help illustrate the concepts of volumetric addition and subtraction. Flat pattern development tools (used in sheet metal design) could be used to create what geometry educators call a "net".

Many high school mathematics educators are already using software applications to help teach in the geometry classroom. Geometer's SketchPad (GSP), for example, is a graphics-based application that utilizes dimension equations to constrain geometry (McClintock, Jiang, & July, 2002; Sanders, 1998). Geometry created in GSP may be readily modified by editing the dimension equations or by dragging on the graphics screen (similar to parametric modeler sketching environments). However, GSP is primarily a 2D application and does not have a robust set of 3D tools. Another application, Cabri® 3D, is a 3-D geometry application designed for use in the mathematics classroom. Cabri® 3D has easy to use 3-D modeling and rendering tools (click and drag), but does not have good dimension equation capabilities. There does not appear to be any software being used by mathematics educators capable of creating fully-rendered 3D geometry based on user-defined equations and mathematical relationships; capabilities that are common to all modern parametric solid modelers.

The purpose of this paper is to describe a study in which Unigraphics NX4, a constraint-based parametric solid modeler, was used in high school mathematics classes to help teach the principles of 3D geometry. NX4 was used in this study because it was readily available for use by the researcher; however no modeling techniques used in the study would preclude the use of other mainstream constraint-based solid modelers. For higher education, the potential benefits of using a solid modeler during high school mathematics instruction are twofold. First, this instructional technique allows high school students to see the modern solid modeling tools used by many engineering professionals. The mathematics classroom, in essence, provides an opportunity to demonstrate engineering tools to prospective students. Second, an improvement in student understanding of geometry concepts would undoubtedly benefit students who enter into engineering-related disciplines.

The objective of this study was measure the extent to which using a parametric solid modeler during high school mathematics instruction affects student learning. Specific research questions addressed in this study were:

1. To what extent does using a parametric solid modeler during high school mathematics instruction affect student understanding of area/volume mathematics principles pertaining to primitive shapes?

2. To what extent does using a parametric solid modeler during high school mathematics instruction affect student ability to apply the mathematics principles pertaining to area and volume to solve unique authentic problems?

STUDY PROCEDURES

This study was conducted in two high schools located in central Illinois. One high school (School A) was a small rural high school while the other (School B) was located in a metropolitan community. The mathematics topic of study dealt with 3D geometric shapes including spheres, cylinders, cones, and prisms. Four intact sections of high school geometry and two intact sections of high school accelerated geometry were used in this study. Three of the sections were used as a control group (n=66) while the remaining three sections were used as an experimental group (n=60). Three cooperating mathematics instructors each taught one experimental and one control section during the study. Refer to Figure 1 for a summary of participant groups.

School/ Instructor	Experimental Group	Control Group
School A Instructor 1	Geometry N=13	Geometry N=21
School B Instructor 2	Accelerated Geometry N=21	Accelerated Geometry N=22
School B Instructor 3	Geometry N=26	Geometry N=23
Total	**60**	**66**

Figure 1. Summary of Participant Groups

The researcher did not create scripted lesson plans for the mathematics instructors to follow during this study. During pre-study meetings, relevant functions of Unigraphics NX4, a constraint-based parametric solid modeling application, were demonstrated and potential mathematics classroom uses for the software were discussed. In the end, each of the mathematics instructors worked with the researcher to develop lesson plans they felt were appropriate for their classes. As a result, specific instructional techniques, homework assignments, and exam items varied between the cooperating instructors. The variation in lesson plans was ideal for this pilot study because it allowed the mathematics instructors to draw upon their prior education and experience working with high school mathematics students while also giving them a sense of ownership of the lessons. The variations also allowed the researcher to explore a variety of techniques for using parametric modeling functions in the mathematics classroom. Deliberate attempts were made to ensure that each instructor was consistent between their respective control and experimental groups. The control and experimental groups spent the same amount of time in the unit of study and both groups completed the same written homework assignments and completed the same post-treatment written exam. Because the researcher operated the software during the experimental treatments with all three instructors, variations in operator ability did not directly influence the study. To help control for contamination between control and experimental groups, student participants were instructed not to discuss their classroom experiences with their peers outside of class.

The control groups received instruction using traditional methods with only the mathematics instructor present in the classroom. The instructional methods used with the control group varied between instructors and typically involved using a white board to sketch geometric shapes and work through mathematics formulae. Several instructors included learning activities that required the use of manipulatives. One such activity used Styrofoam spheres and pre-cut pieces of paper to illustrate the concept of surface area on a sphere. Illustrations in the textbook were also used to present new concepts.

The experimental group received instruction with both the instructor and the researcher present in the classroom. The mathematics instructor presented the mathematics content to the students and was always the primary instructor in the classroom. When prompted by the instructor, the researcher used various modeling techniques to illustrate the concepts being discussed. Frequently the modeling demonstrations had been developed in collaboration with the mathematics instructor prior to class, however impromptu models were created to help answer student questions as well. The use of impromptu models seemed to increase as the study progressed, in part because the mathematics instructors became more comfortable interacting with the software and researcher, and experiences from previous sessions showed the value of using the software in this manner.

At various times during each class period, the researcher worked through problems for the students using Unigraphics NX4. The computer images were projected on a screen for all students to see clearly. Using Unigraphics, the researcher modeled the solid objects using the cognitive modeling strategy outlined by Duncan (1996). The solid modeling techniques used usually involved creating and constraining a two-dimensional sketch which was then extruded or revolved to create a solid. Named expressions were used to constrain the sketches, with the expression names chosen to match the mathematics terminology presented in the text. Boolean operations provided opportunities to illustrate the concept of volumetric addition and subtraction. The solids were shaded, rotated, and sometimes sectioned to help the students visualize the shape. When specific information was required for a calculation (e.g. height and diameter of a cylinder), the dimensions were obtained both algebraically using the expression editor and graphically using various measuring functions in Unigraphics. Several sample demonstrations are discussed in the Sample Demonstrations section of this paper.

Since the topic of 3D solids falls near the end of the high school year each student had accumulated almost a full year of geometry scores prior to this study. The mean scores from the geometry sections were used to measure pre-treatment student performance. Because the objective of this study was to measure the extent to which using a solid modeler during instruction affects student learning with respect to the concepts and application of geometry, traditional geometry examinations were used as assessment instruments in this study. Post-treatment data were collected using geometry content exams comprised of two parts. The first part contained questions designed to measure the extent to which students understand the basic mathematical concepts of geometry and the extent to which they can apply the appropriate mathematical procedures to solve focused problems. The problems given in this part of the exam are similar to problems completed in homework assignments. The second part of the exam consists of test items that are application oriented. These test questions are designed to measure the extent to which students are able to apply mathematical concepts and procedures to solve real word problems. The application oriented questions are problems which have not been previously discussed in class or in the text book. Each cooperating instructor worked with the researcher to create a unique examination that was used with their respective control and experimental groups.

Sample Demonstrations

This section presents examples of the techniques used to present information to the experimental groups during instruction. Because of time constraints, only one of the three control groups was allowed to see the modeling techniques at the conclusion of the study.

Cylinders

When learning about the mathematical formula to define a cylinder, Unigraphics NX4 was used to model a cylinder “parametrically”, which means that the mathematical parameters for the geometric shape “cylinder” were used to define a cylinder. Figure 2 shows the Expression Editor screen of Unigraphics and lists the three parameters used to define a cylinder. The cylinder appeared on the screen as a fully-rendered image that could be dimensioned, dynamically panned, zoomed and rotated (Figure 3A). In this example, dimensions have been added to graphically show the current values for the parameters. The parameters used to create the cylinder were then edited and the graphics image immediately updated to illustrate the relationships between the parameters. In this example, editing the angle of the cylinder from 90° to 75° illustrates the differ-

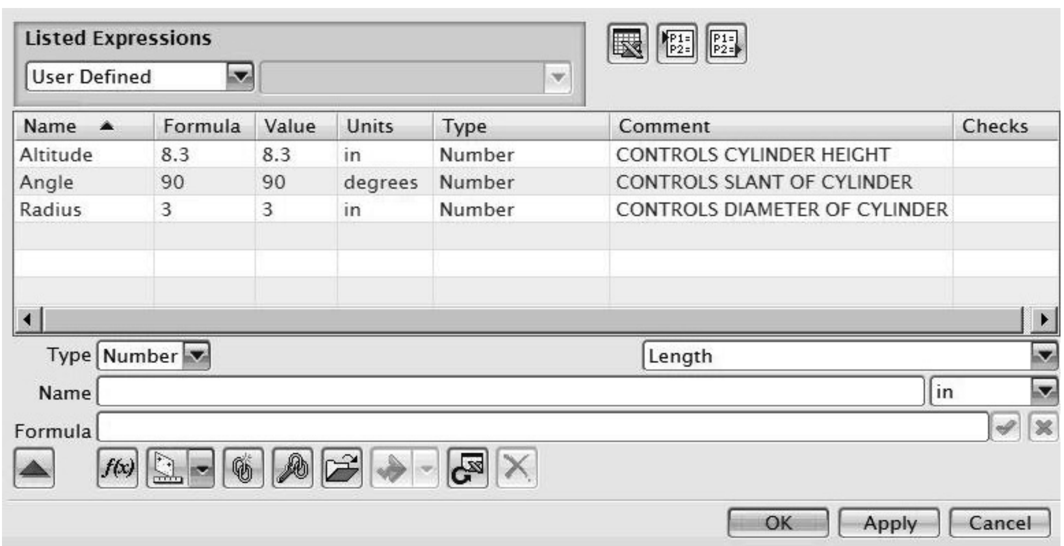

Figure 2. Expression Editor Showing Parameters for a Cylinder

ence between the geometry concepts of altitude and axis length, as well as the difference between right and oblique cylinders (Figure 3B).

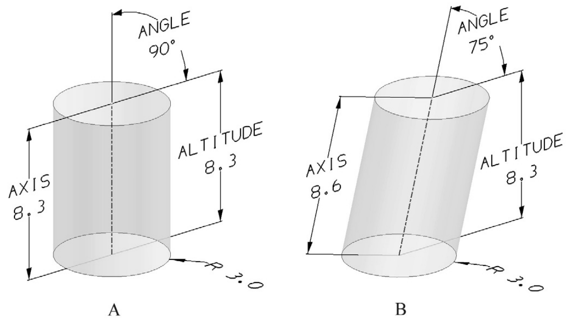

Figure 3. Illustrations of Parametrically Defined Cylinder

Prisms

During the lesson on prisms, students were introduced to the concept of a "net", which in the manufacturing domain is often referred to as a flat pattern. During this lesson the mathematics instructor verbally described what a net was, and asked the researcher to open a previously created model of the right prism seen in Figure 4A. The researcher then used the sheet metal functions of Unigraphics NX4 to create a flat pattern for the prism (Figure 4B). A discussion regarding lateral and overall surface area of the prism and faces followed. The researcher changed various face colors on the solid and net to help the students see the relationship between the 3D solid and the 2D net. The instructor then asked the class to predict if the net would change if the right prism was oblique. Angles 1 and 2 of the prism were then changed and the resulting oblique prism and net were displayed for students to see (Figure 4C and 4D). By adding dimensions to the 3D model, the concepts of altitude and lateral edge were also illustrated. This same model was used to discover how the surface area, lateral surface areas, and volumes compare between right and oblique prisms having congruent bases and a fixed altitude (surface area differs but volume does not). This model was quite helpful because it allowed the mathematics instructor to illustrate many concepts with interaction and input from the students.

RESULTS

In the scores prior to the treatment, the mean course grade for the control group was 84.0% while the mean score for the experimental group was 85.0%. Using a T-Test, it was determined that the 1.0% difference in scores was not statistically significant and the two groups were considered equal for this study. In the post-treatment scores, both the experimental and control group scores were lower than the pre-treatment scores. This drop in scores could be caused by the fact that the pre-treatment scores included all course activities while the post-treatment scores included only exam scores. Also, the topic of 3D geometry may be a more difficult subject which would naturally result in lower examination scores. A comparison of the post-treatment scores shows

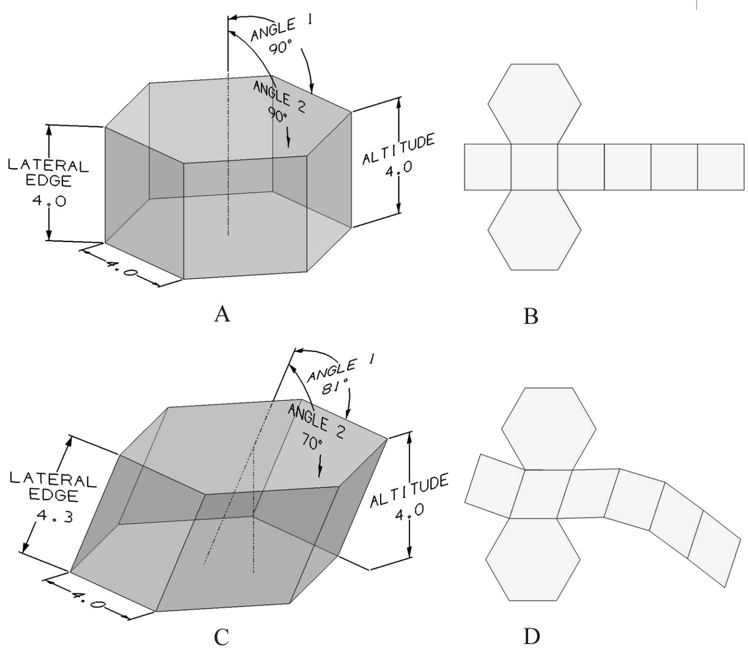

Figure 4. Prisms with Corresponding "Net"

the experimental group was 3.3% to 4.1% higher than the control group in various categories. While there was an observable difference in the means, a T-Test showed the difference in scores between the experimental and control groups was not statistically significant at the .05 level. Figure 5 presents the pre- and post-treatment scores.

Group	Pre-Treatment Course Scores	Post-Treatment Overall Exam Scores	Post-Treatment Geometry Concepts Questions	Post-Treatment Geometry Application Problems
Experimental Group (N=60)	85.0%	84.5%	86.0%	75.7%
Control Group (N=66)	84.0%	81.0%	82.7%	71.6%
Performance Difference Between Groups	1%	3.5%	3.3%	4.1%

Figure 5. Pre- and Post Treatment Scores

The quantifiable measures of this study showed the use of a parametric solid modeler did not significantly change student understanding of geometry concepts or the ability to apply geometry concepts to solve application problems. Although the improvement was not statistically significant, the mathematics instructors commented that they did not know of a parent who would consider a 3% improvement in their child's geometry exam score to be non-significant. The cooperating mathematics instructors were also quick to point out that they observed many non-quantifiable benefits to using the software during geometry instruction.

The instructors commented that the rendering capabilities of the system allowed students to visualize the geometry like never before in their classes. One female student, for example, excitedly told her instructor that for the first time all year she had been able to visualize the geometry concepts being taught in the class. During one class session, after asking her students to complete a problem independently in-class, the instructor quietly asked the researcher to help her "see" the assigned problem by rotating a solid shape on the computer screen. She commented that she has a difficult time visualizing 3D objects and that the software really helped her see the shapes better. Many comments made by the mathematics instructors and students suggest that the realistic rendering and dynamic pan/rotation capabilities of Unigraphics NX4 helped some students visualize three dimensional objects more readily. This is worthy of note because the importance of visualization in mathematics instruction is receiving attention in the mathematics literature (Sanders, 1998) and the National Council of Teachers of Mathematics (NCTM) is placing a renewed emphasis on developing spatial visualization skills of geometry students (McClintock et al., 2002).

The mathematics instructors also commented that the solid modeling software allowed them to test their students' understanding of geometry principles by asking probing questions they would not normally be able to answer using graphical means. The cooperating mathematics instructors were also very pleased to be able to teach geometry to their students using real-world applications. They liked the idea of breaking away from the textbook to show how geometry may be used in engineering graphics. One instructor commented that even the simple things like using ASME dimensioning practices in homework problems brought a real world flavor to the topic; something that is lacking in many high school mathematics classrooms.

CONCLUSIONS

This paper describes a pilot study in which Unigraphics NX4, a constraint-based parametric solid modeler, was used during instruction in high school geometry classes. Although students in the experimental group did have higher post-treatment scores, statistical analysis performed on pre- and post-treatment student performance data suggests that using a parametric modeling does not significantly affect student learning of geometry. However, feedback from the three cooperating mathematics instructors, as well as comments made by many of the students, suggests that there are several potential benefits of using a parametric solid modeler during high school mathematics instruction.

The cooperating mathematics instructors appreciated the opportunity to show their students real-world applications for mathematics. This is important because educational researchers have long realized the importance of context in the learning environment, and the lack of an authentic context for learning experiences has long been a concern mathematics education (Hiebert & Lefevre, 1986; Silver, 1986). Exposure to real world applications of math and science may also help students see value in pursuing STEM (Science, Technology, engineering and Mathematics) related education (Kesidou & Koppal, 2004; Raju, Sankar, & Cook, 2004; Swift & Watkins, 2004). Finally, the mathematics students and instructors made many comments regarding the value of being able to visualize the 3D shapes using the solid modeling software.

Because this study had a small number of student participants, the reliability and generalizability of findings is limited. The cooperating mathematics instructors have invited the researcher to continue this study next year at their schools, and based on information gleaned from this pilot study several modifications to the study procedures will be made. The researcher will work with the cooperating mathematics instructors to develop common lesson plans, demonstrations, and pre- and post-treatment measurement instruments that will be used by future participants in the study. Furthermore, because comments made by the participants of this pilot study suggest that improved visualization is a significant potential benefit obtained from using a solid modeler during mathematics instruction, the scope of future work will be expanded to explore this phenomenon. Finally, this pilot study did not afford students the opportunity to use a solid modeler themselves. Future work will explore the feasibility and potential benefits of allowing geometry students to use a parametric solid modeler during mathematics instruction.

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