Simulations of Carnival Rides and Rube Goldberg Machines for the Visualization of Concepts of Statics and Dynamics

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Abstract

Solid modeling is widely used as a teaching tool in summer activities with high school students. The addition of motion analysis allows concepts from statics and dynamics to be introduced to students in both qualitative and quantitative ways. Two sets of solid modeling projects – carnival rides and Rube Goldberg machines – are shown to allow the students creative freedom while challenging them to understand the physics of the simulated motion. Possible benefits of including similar motion simulations into engineering classes as exercises or in-class demonstrations are discussed.

INTRODUCTION

Solid modeling is now widely used in most engineering and engineering technology programs. It has also become a common tool for workshops and seminars for middle and high school students. The basic functions of modern solid modeling software are relatively easy to learn, and most students find the learning experience to be fun. As such, solid modeling is an excellent tool for stimulating interest in engineering and technology careers.

While solid modeling exercises help students develop important three-dimensional visualization skills and can be used as an introduction to the engineering design process, the addition of analysis and simulation tools can greatly expand their value. In the past, the authors have reported on the use of finite element analysis in a design-analyze-build-test experience for high school students (Howard and Williams, 2007), as well as the inclusion of some simple motion analysis exercises with another group of students (Howard et al., 2009).

This paper focuses on motion analysis studies performed by students in the Summer Ventures in Science and Mathematics program during 2008 and 2009. During the last week of the program, students work independently on a project of their choosing. Several students chose to model carnival rides or Rube Goldberg machines as their projects. Some of the concepts explored by the students required them to learn some basic concepts from statics and dynamics. In addition to reporting on these projects, this paper will examine the potential of using similar exercises in engineering mechanics classes.

THE SUMMER VENTURES PROGRAM

The Summer Ventures in Science and Mathematics program was established in 1984 by the North Carolina General Assembly. The program is open to rising high school juniors and seniors who are state residents. Selected participants spend four weeks at one of six University of North Carolina System campuses. At East Carolina University, students select three areas of study for two-hour class sessions during the first three weeks, and then choose one area for independent study the fourth week. Engineering was added as a study area in 2005. During the first three weeks, engineering activities are almost evenly divided between solid modeling and robotics. In the solid modeling sessions, students learn basic modeling skills using

SolidWorks software, use finite element analysis and rapid prototyping to improve the design of a structural part, and perform some simple motion analysis of a four-bar linkage.

Over the past two summers, the number of students choosing solid modeling-related and robotics-related projects have been approximately equal (some students choose projects that combine both, using solid modeling to design components, which are built on a rapid prototyping system, for their robots). While some students have a clear idea for their project, most need suggestions to start. We have added carnival rides and Rube Goldberg machines as suggested projects, and have found that these activities allow students to exercise unlimited creativity, while serving as excellent teaching tools for students to learn concepts of engineering mechanics.

MOTION ANALYSIS SOFTWARE

Assemblies are constructed in the SolidWorks program by bringing in individual components and adding constraints (referred to as "mates") to restrict the relative motion between the components. These mates define joints in motion analysis. For example, the four-bar linkage in Figure 1 includes mates forcing the flat surfaces of adjacent links to be coplanar, and mates forcing the holes to be concentric. The resulting allowable motions are those of revolute (hinge) joints, with each joint having a single degree of freedom. In a motion analysis, a motor can be applied to drive one of the links, and the other links will move consistent with the constraints of the revolute joints.
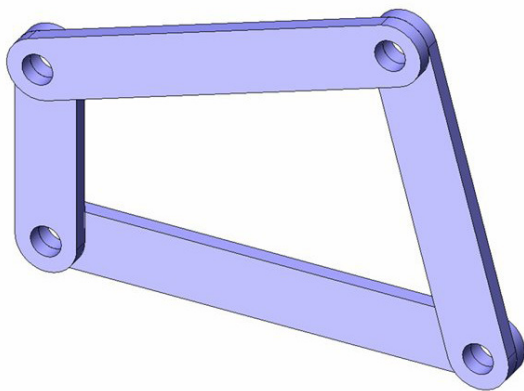

Figure 1. Four-Bar Linkage Model

The SolidWorks 2009 software contains three options for motion simulation and analysis:

- Animation: allows for motors to be specified and the resulting motions calculated. The motion of the four-bar linkage shown in Figure 1 can be simulated with this option.

- Basic Motion: allows for the addition of forces (including gravity), springs and dampers, and solid body contacts (without friction or impact properties). Animation and Basic Motion are part of the core SolidWorks software.

- Motion Analysis: allows for friction and impact properties to be added to solid body contacts, and for quantitative display of results such as velocities, acceleration, and forces. This option is available only if the SolidWorks Motion add-in program is installed and loaded. SolidWorks Motion is included in the SolidWorks Education Edition and Student Edition, but not in the SolidWorks Student Design Kit that is included with many textbooks. (Note that prior to the 2009 release of the SolidWorks Software, the three options listed were referred to as Assembly Motion, Physical Simulation, and COSMOSMotion, respectively. Although the work reported in this paper was performed using SolidWorks 2007 and 2008, we have chosen to use the current nomenclature.)

As students begin to experiment with motion analysis, they quickly learn that accurately simulating physical events that appear to be simple can in fact be extremely complex and computationally challenging. This is especially true if solid body contacts are included. When setting up an analysis, the user can specify the number of time steps for calculations, the degree of accuracy required for contact calculations, and the overall accuracy tolerance for the analysis. Often, a simulation will produce unexpected results or will not be completed, and these parameters, or input properties such as friction and impact characteristics, will need to be adjusted.

CARNIVAL RIDES

As an introduction to motion analysis, students were stepped through an analysis of the simple "teacup" ride illustrated in Figure 2.
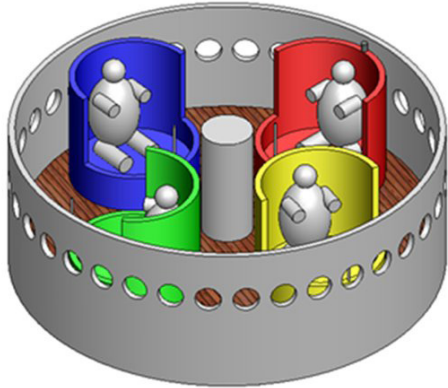

Figure 2. Teacup Ride

The component models are provided to the students, so that the exercise focuses on assembly and motion analysis of the ride. A motor is added to cause the floor of the ride to rotate relative to the fixed base, and other motors cause the seats to rotate relative to the floor. The ride can be simulated using only the Animation option, but to view numerical results, the SolidWorks Motion add-in must be enabled.

The results that are of interest here are the velocity and acceleration that are experienced during the ride. Since these parameters are displayed for the center of mass of a component, a small cylindrical object (a "sensor") is added to one of the seats immediately behind the head of the rider. A graph of the velocity magnitude is shown in Figure 3. For this analysis, the floor was set to rotate at 15
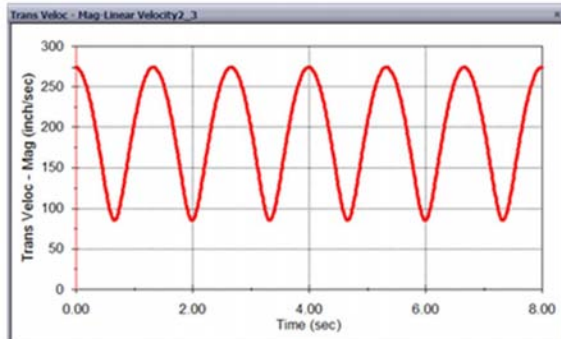

Figure 3. Velocity of Sensor in Teacup Ride

rpm, and the seat was set to rotate at 45 rpm relative to the floor (both rotations were clockwise).

The results of this analysis are easy to confirm with hand calculations, and introduce students to the concepts of curvilinear motion, relative motion, and vector quantities. If the center of the seat is called point A and the location of the sensor is called point B, then the velocity of point B can be written as:

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

Equation 1.

where $\mathbf{v}_{B/A}$ is the velocity of point B relative to point A. For a point traveling in a circular path, the magnitude of velocity is the angular velocity ω times the radial distance from the center of rotation to the point of interest, so the magnitude of point B's velocity can be written as:

$$\mathbf{v}_B = \omega_{Floor} r_{OA} + \omega_{Seat} r_{AB}$$

Equation 2.

where point O is the center of the ride. It is worth pointing out to students that Equation 2 is valid only if the two velocity components are along the same line of action. The more general form of the velocity equation is written with vector cross-products, but teaching vector mathematics to the high school students is not practical in the time available. However, introducing the *concept* of a vector quantity is important. The radial distance r_{OA} is 60 inches, and the distance r_{AB} is 28.5 inches. The angular velocity of the floor is 15 rpm (which converts to $\pi/2$ radians per second). When entering the value for the angular velocity of the seat, the tendency for students is to enter 45 rpm ($3\pi/2$ radians per second). However, this is the value of the angular velocity of the seat relative to the floor. To find the angular velocity of the seat, this relative velocity must be added to the velocity of the floor:

$$\omega_{Seat} = \omega_{Floor} + \omega_{Seat/Floor}$$

Equation 3.

Therefore, the magnitude of the angular velocity of the seat is equal to:

$$\omega_{Seat} = \pi/2 + 3\pi/2 = 2\pi \, rad/s$$

Equation 4.

The maximum velocity occurs when the sensor is at its maximum distance from the center (when the rider is facing inward, as in Figure 4), since the two velocity components of Equation 2 are associated with velocities in the same direction. Therefore, the maximum magnitude of the velocity is:

$$v_{Bmax} =$$
$$\left(\frac{\pi}{2}\frac{rad}{s}\right)(60in) + \left(2\pi\frac{rad}{s}\right)(28.5in)$$
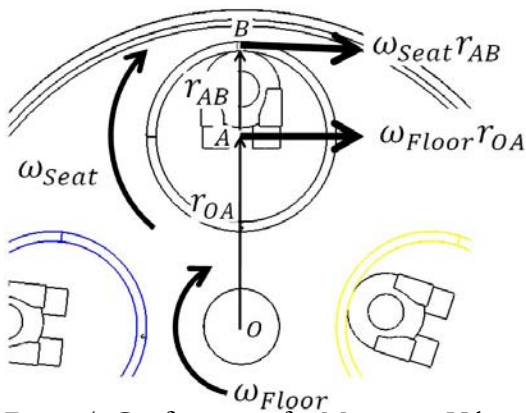$$= 273\frac{in}{s}$$

Equation 5.

Figure 4. Configuration for Maximum Velocity

This value agrees with the value from the simulation, as shown on the graph in Figure 3. To find the minimum value of velocity magnitude, we note that when the sensor box is at its minimum distance from the center, as in Figure 5, then the two velocity components are in opposite directions. If we define the positive direction as to the right (clockwise relative to the center of the ride), then the velocity at this point is:

$$v_{Bmin} =$$
$$\left(\frac{\pi}{2}\frac{rad}{s}\right)(60in) - \left(2\pi\frac{rad}{s}\right)(28.5in)$$
$$= -85\frac{in}{s}$$

Equation 6.

This value also agrees with the simulation result of Figure 3, although since the magnitude of velocity is graphed, the negative sign is dropped.

A similar set of calculations can be made for the extreme values of acceleration magnitude.
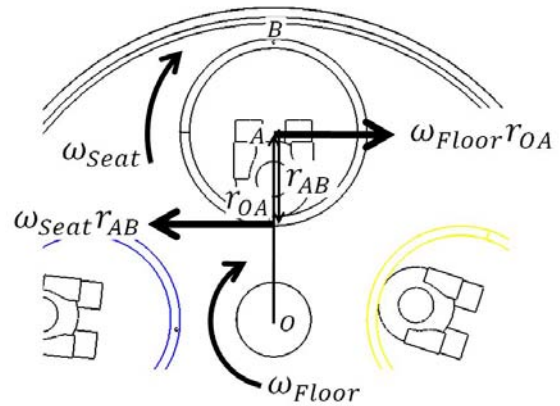

Figure 5. Configuration for Minimum Velocity

Although the calculations for this example are simple, there are several important points that will be new for most high school students:

- The concept that velocity and acceleration are vector quantities, with both magnitude and direction, and can be added algebraically as in Equations 5 and 6 only when directions are along the same line of action.

- The concept of relative motion, which can be introduced by having students discuss the apparent motion of other cars passing them in both directions when they are traveling in a car on a straight road.

- The fact that a body traveling in a circular path must have acceleration, even if the angular velocity is constant, since the velocity is constantly changing direction. Since force is proportional to acceleration, this means that force is necessary to cause a body to follow a curved path. An easy to understand analogy is a car making a turn on a slick road. If the wheels begin to slide, then the car will travel in a straight line off the road. To travel the curved path, the friction between the tires and road must provide a force toward the center of curvature.

- The fact that even relatively low rotational speeds can produce high accelerations. In this example, the seats do not appear to be spinning at an unreasonably high speed,

but the maximum acceleration is 1273 in/s², or 3.3g. Students usually have a vague understanding of g-forces, and those who choose to model carnival rides are encouraged to research the g-forces experienced during actual rides and the effects of g-forces on the body.

During the Summer Ventures 2009 program, three students chose to model carnival rides as their final projects. Figure 6 shows a "Yo-Yo" ride which was designed by one of the students. In the model, the center shaft spins and the seats are propelled in and out radially by only gravity and inertial forces.
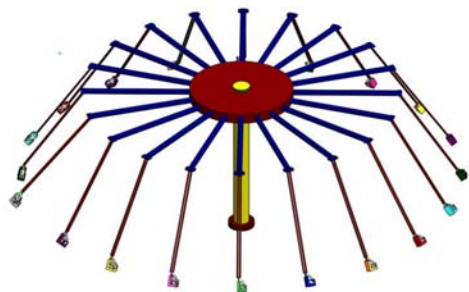
Figure 6. Yo-Yo Ride Model

In the simulation of this ride, it was noted that applying a constant-speed motor to the center shaft produced wild oscillations of the hanging seats. In order to produce a realistic simulation, the speed needed to be ramped up slowly to avoid the sudden change of acceleration (introducing the concept of jerk, the rate of change of acceleration), and friction needed to be added to the joints at the top of each seat's hanger. The student also varied the mass properties of the seat/riders to study the effects of the mass on the accelerations.

The rate of acceleration and the friction properties were also important in the simulation of the student-designed Ferris Wheel shown in Figure 7. A Ferris Wheel is not usually thought of as a "thrill ride," and so the accelerations experienced by riders were expected to be low. If the rotational velocity was applied instantaneously and friction was omitted from the model, then the calculated accelerations were surprisingly high. The student was able to isolate the contributions of the rota-

tional speed and the swinging of the seats on overall acceleration and show that if the swinging is minimized with friction and gradual changes of rotational speed, then g-forces on Ferris Wheel riders are indeed low.
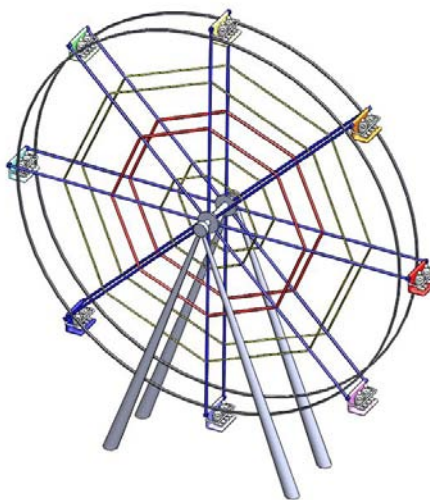
Figure 7. Ferris Wheel Model

The third ride modeled by a student is one that goes by various names ("Viking Ship" is one common name) and was called the "Pirate Ride" by the student. The ride, which is illustrated in Figure 8, rocks the passengers back and forth in gradually larger oscillations until finally completing a complete 360-degree loop, often holding the riders up-side down momentarily.

Figure 8. Pirate Ride

The student modeling this ride noted that the motion of the ride is controlled so that it appears that the ride swings higher and higher under its

own momentum. Therefore, this exercise presents a good opportunity for introducing the concept of conservation of energy. A motor must be present to enable the ride to swing higher than its previous high point. The student investigated the torque necessary to drive the ride. In doing so, the purpose of the large counterweight and the effects of friction were seen. The effects of friction on all of these rides can lead to a discussion of friction-reduction technologies (bearings and lubrication).

While these projects were completed individually, the three students designed their rides to scale to a common rider size, and put their designs together into a virtual amusement park, as shown in Figure 9.

Figure 9. Virtual Amusement Park

It should be noted that during the first three weeks of the program, students received about six hours of instruction in using the SolidWorks software, and spent about four more hours working independently to redesign a structural part. During the final week, while the students had a total of six days to complete their projects, they were required to write a paper and to make an oral presentation. Therefore, most of the project work was completed in three to four days. The complexity of the models completed during the program is a good indication of how quickly students can learn solid modeling, especially considering the amount of time that must be devoted to the addition of motion to the models.

RUBE GOLDBERG MACHINES

Rube Goldberg was an engineer and a popular cartoonist of the early 20th century, and was best known for his depictions of machines designed to perform a simple task with a complex series of steps. His name is now synonymous with any design that is needlessly complicated. High school students may be familiar with Rube Goldberg machines from Rube Goldberg contests held around the country for student teams, or from the popular board game Mouse Trap from Hasbro, Inc.

The student design shown in Figure 10 includes several steps that can be simulated with the Solid-Works Motion program:

- Balls and rollers rolling down ramps,

- A series of dominoes falling over,

- A balanced lever which rotates when the balance is upset by a ball falling on one end,

- A "water wheel" type mechanism that rotates when a ball hits it,

- A gear train that transmits torque and modifies rotational speed, and

- A cam that lifts an object, in this case a sign with a message that is hidden until the final step of the simulation.
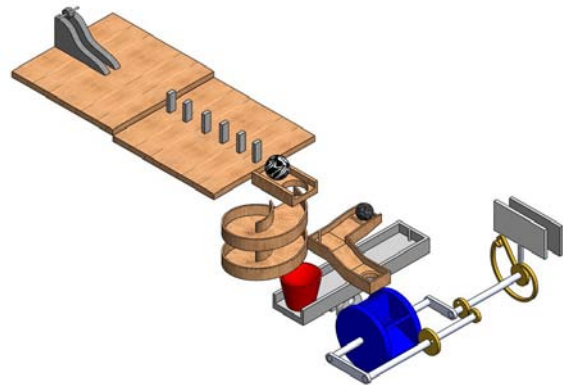

Figure 10. Rube Goldberg Machine Model

The simulation of the carnival rides involved adding motors and friction to joints that were created from assembly mates. Most of the elements of Rube Goldberg machines involve the specification of solid body contacts, which make the simulation more computationally complex.

When using SolidWorks simulation, a physical event can be accurately replicated only if all forces acting on a body are accounted for in the model. This makes the simulations a good tool for introducing concepts from mechanics, beginning with the free body diagram. We examine in detail two common elements of Rube Goldberg machines in some detail – a roller moving down a ramp and a series of falling dominos.

Consider the model of a roller on a ramp shown in Figure 11, and the free body diagram of the roller shown in Figure 12.
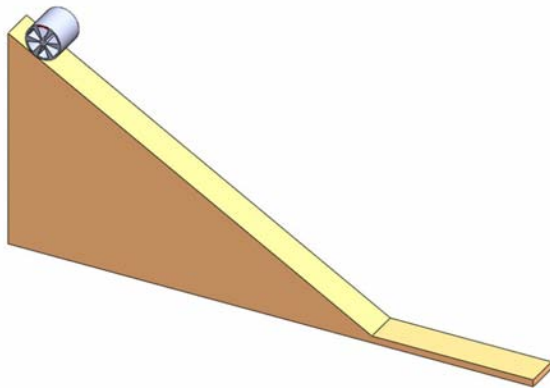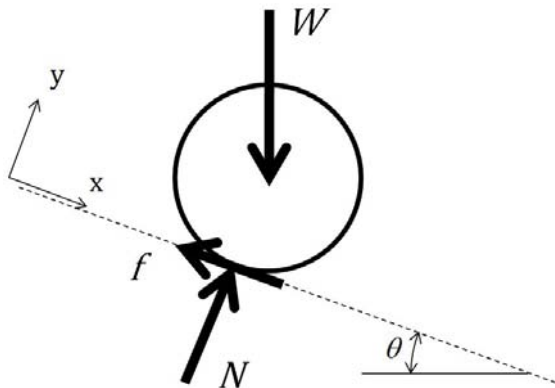

Figure 11. Ramp and Roller Model

Figure 12. Free Body Diagram of Roller

The forces shown are included in the model by specifying gravity (for W, the weight) and contact between the roller and ramp (for N, the normal force). The definition of the contact includes specification of the coefficient of friction (for f, the friction force) and impact properties.

As a first trial of the simulation, students may begin with no friction. Of course, the roller will slide down the ramp with no rotation. The magnitude of the velocity of the center of mass of the roller is displayed in Figure 13.
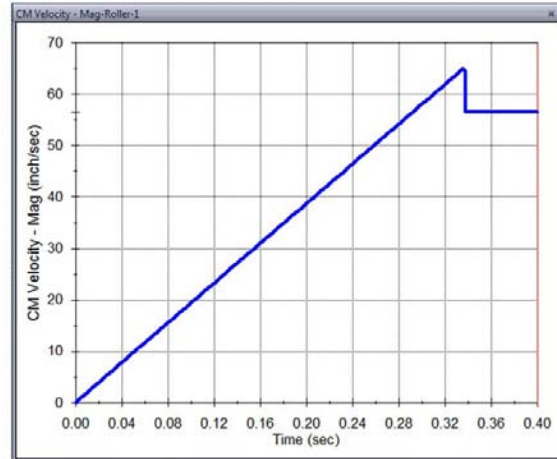

Figure 13. Velocity of Roller - No Friction

The velocity of the roller when it reaches the bottom of the ramp, 65 inches per second, can be calculated by equating the initial potential energy (the initial height, 5.5 inches, times the mass times the gravitational acceleration, 386 in/s^2) and the final kinetic energy (1/2 times mass times velocity squared), and solving for the velocity. Students can change the mass properties of the roller and verify that the final velocity is independent of the roller's mass. When the roller makes contact with the flat part of the ramp, then a reduction in velocity is observed. The magnitude of this reduction is dependent on the impact properties input for the contact between the roller and ramp. These impact properties will be discussed later.

With friction added, the equilibrium equations for the "no slip" condition can be written as:

$$\Sigma F_x = W\sin\theta - f = ma_x$$

Equation 7.

$$\Sigma F_y = N - W\cos\theta = 0$$

Equation 8.

and

$$\Sigma M_c = -fr = I_c \alpha$$

Equation 9.

where the x-axis is parallel to the surface of the ramp, I_c is the mass moment of inertia about the centroidal axis, and α is the angular acceleration

of the roller. It is important to note that if there is no slip, the acceleration a_x and the angular acceleration α are related:

$$a_x = r\alpha$$

Equation 10.

The angular acceleration can be found by substituting Equation 10 into Equation 7 and solving Equations 7 and 9 simultaneously:

$$\alpha = \frac{W\sin\theta r}{I_c + mr^2}$$

Equation 11.

Slippage can be visualized in the motion analysis with the addition of a "trace path" for a point on the perimeter of the roller. A trace path for a high friction coefficient is shown in Figure 14. (Note that a radius has been added to the bottom of the ramp to eliminate the collision between the roller and the flat part of the ramp.)
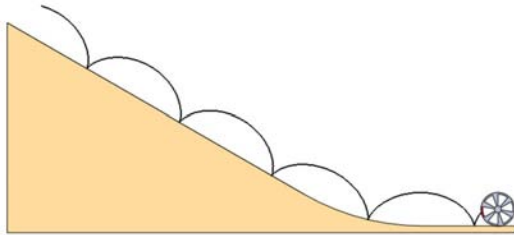

Figure 14. Trace Path with High Friction Coefficient

When the point on the roller makes contact with the ramp, the velocity goes to zero if there is no slippage. The sharp point on the trace path is indicative of zero velocity; a plot of velocity vs. time can also be generated to verify that no slippage occurs.

When a low friction coefficient is specified, a trace path such as the one shown in Figure 15 is generated. Note that when the point on the roller is adjacent to the ramp, the smooth curve of the trace path indicates that the velocity of the point does not go to zero.

It is helpful to refer to the free body diagram of Figure 12. For students to understand this problem, they need to understand that Equations 7 and 8 are sufficient to describe the motion of a particle (body with all forces acting through a
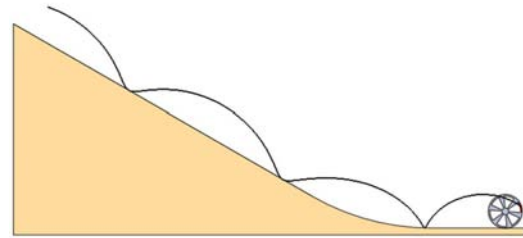

Figure 15. Trace Path with Low Friction Coefficient

single point). For a rigid body, Equation 9 must also be included. When any non-zero friction force is present, then the roller behaves as a rigid body. The friction force causes the sum of moments about the center of the roller to be non-zero and therefore angular acceleration will occur.

With sufficient friction present to prevent slipping, the velocity of the roller at the bottom of the ramp can be determined by integrating the angular acceleration found from Equation 11 to find the angular velocity.

When performing the simulations, students may notice that the roller appears to penetrate the ramp at times. This observation can lead to a discussion of how the program handles contacts between components.

Contact properties must be input for every pair of bodies that can come into contact in an analysis. If no contact properties are entered, then the bodies will simply pass through each other. With contact enabled, then at each time step of the analysis, the positions of the bodies are evaluated to determine if contact has occurred. Since the bodies are assumed to be elastic, some penetration is allowed between the bodies. As a result of the penetration, a force is generated that tends to push the bodies apart. In effect, the contact is simulated as a spring between the two bodies, with the stiffness and damping properties of the spring specified in the contact definition. Calculation of these properties is difficult, and so the default properties of the software are normally used. This leads to an important concept regarding engineering analysis: any model of a real-world event includes approximations. The accuracy of the model is affected by

the approximations, and engineering judgment is required in the formulation of the model and the interpretation of the results. In the case of the roller on the ramp, the contact properties provide an approximation of the rolling resistance of the system. Unless the final speed needs to be calculated precisely, the default properties represent a reasonable approximation of energy loss during rolling.

The simulation of falling dominos is more complex than most other elements of a Rube Goldberg machine. In order to accurately simulate the dominos' behavior, several solid body contacts must be specified. To help students understand the mechanics of the problem, the free body diagram shown in Figure 16 is helpful. The force from the object colliding with the domino is labeled F. When the domino is on the verge of tipping over, the normal force will be concentrated at the corner labeled O.
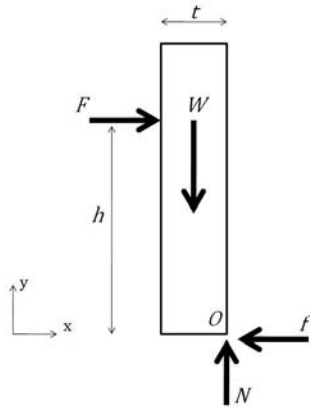

Figure 16. Free Body Diagram of a Domino

The equilibrium equations for the domino are:
$$\Sigma F_x = F - f = 0$$
Equation 12.
$$\Sigma F_x = N - W = 0$$
Equation 13.
$$\Sigma M_O = W\left(\frac{t}{2}\right) - Fh = 0$$
Equation 14.

From these equations, and the maximum friction value of $\mu_s N$, we find that the force required for the domino to slip along the surface is:
$$F_{slip} = f_{max} = \mu_s N = \mu_s W$$
Equation 15.

while the force required for the domino to tip is:
$$F_{tip} = \frac{Wt}{2h}$$
Equation 16.

Comparing Equations 15 and 16, we see that if the domino is to tip (which is the desired action to begin the chain reaction of domino movements) rather than slip, then:
$$\frac{t}{2h} < \mu_s$$
Equation 17.

The significance of Equation 17 is that if the force is applied toward the bottom of the domino (low value of h) and/or the friction coefficient is small, then the condition of Equation 17 will not be satisfied and the domino is more likely to slip rather than to tip. A simulation of this condition is shown in Figure 17. Note that while the initial motion is slipping, the friction force between the spinning roller and the domino causes a downward force on the face of the domino, resulting in a moment that causes the domino to tip "backwards". (This is a good example of a dynamic event that is difficult to describe in words or with still images, and can be more easily understood with an animation.)
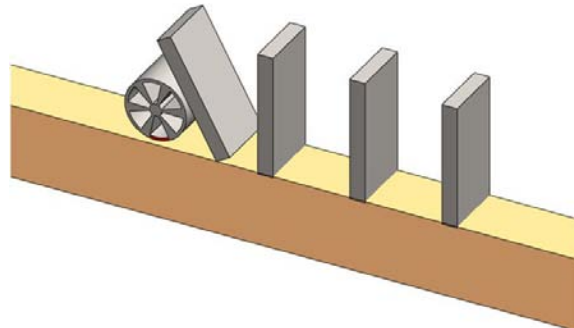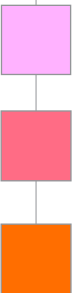

Figure 17. Domino Simulation with Low Contact Point, Low Friction Coefficient

By adding a step down to the ramp so that the roller contacts higher on the domino (thus increasing h), and by increasing the friction coefficient, the desired action can be simulated (using the same roller and domino geometry), as in Figure 18.
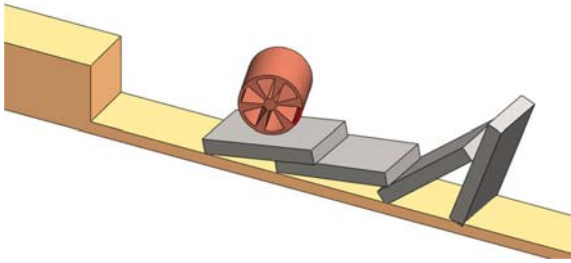
Figure 18. Domino Simulation with High Contact Point, Moderate Friction Coefficient

As noted earlier, the action of the dominos is more difficult to simulate than most other elements in a Rube Goldberg machine. There must be solid body contacts specified between each domino and the base, between any two dominos expected to make contact, and between the body providing the initial contact and the first domino. Changing the friction or impact properties of any of these contacts will affect the entire simulation, sometimes producing unexpected results. The relative weights of the roller and the dominos are also important, as is the velocity with which the roller strikes the first domino (a light roller traveling at low speed can bounce off of the first domino, while a heavy roller traveling at high speed can continue moving through the entire row of dominos rather than initiating the desired chain reaction). While the motion can be visually simulated by adding constraints that are not representative of the actual action of the dominos (such as adding hinge-type joints between the front edge of each domino and the base), a much richer learning experience is gained by attempting to include realistic forces and constraints and investigating the effects of changing their parameters.

As with carnival rides, Rube Goldberg machines allow for individual projects to be incorporated into a team effort. Team members can agree on the overall layout, specify the interfaces between elements, design and test their individual portions and then assemble and simulate the entire machine.

DISCUSSION

Quantitative assessment of the effectiveness of these exercises in helping students understand mechanics concepts is not particularly meaningful because of the small number of students who have been involved to date. Also, in the Summer Ventures program, we are fortunate to work with students who are both high achievers academically and who have expressed interest in math- and science-related fields. However, evaluation of the students' reports and presentations leads us to believe that the students enjoyed the exercises and learned about engineering design and analysis.

While there is no doubt that visualization of problems is a stumbling block for many students in statics and especially in dynamics, incorporating demonstrations and/or student projects into these classes is often difficult to do because of time constraints. Also, many multi-media educational tools have been developed over the past couple of decades, with mixed success. Dillon and Gabard (1998) presented an evaluation of the use of hypermedia in education and concluded that most claims of educational benefits could not be supported. However, well-planned and delivered exercises, such as those reported by Philpot et al. (2005) have been shown to significantly help student understanding of difficult concepts in mechanics.

Crouch et al. (2004) conducted demonstrations in introductory physics classes and measured their effectiveness with an end-of-semester test. They found that the demonstrations themselves resulted in little improvement in student comprehension. However, when the demonstration was preceded by the students making predictions as to what would happen, significant improvements were seen. For in-class demonstrations, it seems to be an effective use of class time to allow students a few minutes to predict the outcome. The projects completed by the students in the summer sessions were consistent with this approach. Since they knew the desired result before beginning a simulation, they were required to compare the simulation result to an expected outcome, and were forced to consider the factors that could have led to an unexpected result. If these exercises are adapted for in-class use, then allowing time for students to think about the problem, draw a free body diagram, and

predict the action of the simulation is expected to increase the effectiveness of the activity.

CONCLUSIONS

Carnival rides and Rube Goldberg machines have been shown to be solid modeling projects which allow high school students to exercise creativity and to learn about the physics of mechanics in a fun environment. Some components of these exercises are of potential use as lab projects or in-class demonstrations in statics and dynamics classes.

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